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(Pre-CBCS)

(4th Semester)

MATHEMATICS

Paper : MATH–241

(Vector Calculus and Solid Geometry)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Tick (✓) the correct answer in the brackets provided :

1. If $\vec{a}$ and $\vec{b}$ are two mutually perpendicular proper vectors, then $\vec{a}$ $(\vec{b}$ $\vec{a})$ is parallel to

(a) $\vec{a}$ ( )

(b) $\vec{b}$ ( )

(c) $\vec{a}$ $\vec{b}$ ( )

(d) None of the above ( )

2. If $|\vec{a}|$ 4, $|\vec{b}|$ 5 and $\vec{a}$ $\vec{b}$ 0, then $\vec{a}$ $\vec{b}$ is

(a) $20\hat{n}$ ( )

(b) $9\hat{n}$ ( )

(c) $\hat{n}$ ( )

(d) 0 ( )

3. The vector $\vec{V} \quad (\ 4x \quad 6y \quad 3z)\hat{i} \quad (\ 2x \quad y \quad 5z)\hat{j} \quad (5x \quad 6y \quad az)\hat{k}$ is solenoidal, then the value of a is

(a) 5 ( )

(b) 8 ( )

(c) 3 ( )

(d) None of the above ( )

4. Suppose V be the volume bounded by a closed surface S, $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ and $\hat{n}$ is the unit vector normal (outward) to the surface S, then

$$\vec{r} \quad \hat{n}dS$$
$$S$$

is

(a) 0 ( )

(b) $4V$ ( )

(c) $2V$ ( )

(d) $3V$ ( )

5. The equation of pair of straight lines through the origin perpendicular to the pair $ax^2 \quad 2hxy \quad by^2 \quad 0$ is

(a) $ax^2 \quad 2hxy \quad by^2 \quad 0$ ( )

(b) $bx^2 \quad 2hxy \quad ay^2 \quad 0$ ( )

(c) $ax^2 \quad 2hxy \quad by^2 \quad 0$ ( )

(d) $bx^2 \quad 2hxy \quad ay^2 \quad 0$ ( )

6. If the equation $ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$ represents a circle, if

(a) $ab \quad h^2 \quad 0$ ( )

(b) $ab \quad h^2 \quad 0$ ( )

(c) $a \quad b$ and $h \quad 0$ ( )

(d) $a \quad b \quad 0$ ( )

7. The intercepts made on the axes by the plane $3x \quad 4y \quad 6z \quad 12 \quad 0$ are

(a) 4, 3 and 2 ( )

(b) 4, 3 and 5 ( )

(c) 5, 7 and 9 ( )

(d) None of the above ( )

8. The shortest distance between the line $\frac{x \quad 1}{4} \quad \frac{y \quad 2}{3} \quad \frac{z \quad 3}{1}$ and z-axis is

(a) $\frac{12}{5}$ ( )

(b) $\frac{11}{\sqrt{5}}$ ( )

(c) $\frac{11}{5}$ ( )

(d) $\frac{12}{7}$ ( )

9. The equation of sphere which passes through the origin and makes equal intercepts of unit length of the axes is

(a) $x^2 \quad y^2 \quad z^2 \quad 1$ ( )

(b) $(x \quad 1)^2 \quad (y \quad 1)^2 \quad (z \quad 1)^2 \quad 0$ ( )

(c) $x^2 \quad y^2 \quad z^2 \quad x \quad y \quad z \quad 1$ ( )

(d) $x^2 \quad y^2 \quad z^2 \quad x \quad y \quad z \quad 0$ ( )

10. The condition that the plane $lx \quad my \quad nz \quad 0$ touches the cone $ax^2 \quad by^2 \quad cz^2 \quad 0$ is

(a) $bcl^2 \quad cam^2 \quad abn^2 \quad 0$ ( )

(b) $al^2 \quad bm^2 \quad cn^2 \quad 0$ ( )

(c) $a^2l \quad b^2m \quad c^2n \quad 0$ ( )

(d) None of the above ( )

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

State *True* or *False* by putting a Tick (✓) mark in the brackets provided and give a brief justification :

1. If $\vec{a} \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k}$, $\vec{b} \quad \hat{i} \quad 3\hat{j} \quad 5\hat{k}$ and $\vec{c} \quad \hat{i} \quad \hat{j} \quad 6\hat{k}$, then the value of $\vec{a} \quad (\vec{b} \quad \vec{c})$ is 5.

True ( ) *False* ( )

Justification :

2. The value of $\frac{1}{r}$, where $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ is $\frac{\vec{r}}{r^3}$.

True ( ) *False* ( )

Justification :

3. The equation of the diameter of the conic $4x^2 \quad 6xy \quad 5y^2 \quad 1$ conjugate to the diameter $y \quad 2x \quad 0$ is $10y \quad 7x \quad 0$.

True ( ) *False* ( )

Justification :

4. The equation of the plane through the line $x \quad y \quad z \quad 3 \quad 0,\ 2x \quad y \quad 3z \quad 1 \quad 0$ and parallel to the line $\frac{x}{1} \quad \frac{y}{2} \quad \frac{z}{3}$ is $x \quad 5y \quad 3z \quad 7 \quad 0$.

True ( ) *False* ( )

Justification :

5. The equation of the orthogonal projection of the curve $2x \quad y \quad z \quad 3$, $x^2 \quad 2y^2 \quad 3z^2 \quad 1$ on the z-plane is

$$z \quad 0,\ 13x^2 \quad 5y^2 \quad 36x \quad 18y \quad 12xy \quad 26 \quad 0$$

True ( ) *False* ( )

Justification :

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

Unit—I

1. *(a)* Find a unit vector perpendicular to the plane of $\vec{A} \quad 2\hat{i} \quad 6\hat{j} \quad 3\hat{k}$ and $\vec{B} \quad 4\hat{i} \quad 3\hat{j} \quad \hat{k}$. 3

(b) If ABC be a triangle, then prove that

$$\cos A \quad \frac{b^2 \quad c^2 \quad a^2}{2bc}$$ 3

(c) Find the set of vectors reciprocal to the set $\hat{i} \quad 2\hat{j} \quad 3\hat{k}$, $5\hat{i} \quad \hat{j} \quad \hat{k}$ and $\hat{i} \quad \hat{j} \quad \hat{k}$. 4

2. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then prove that $[\vec{a} \quad \vec{b} \; \vec{b} \quad \vec{c} \; \vec{c} \quad \vec{a}] \quad [\vec{a} \; \vec{b} \; \vec{c}]$. 5

(b) If three concurrent edges of a parallelepiped is given by $\vec{a} \quad 2\hat{i} \quad 3\hat{j} \quad 4\hat{k}$, $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$ and $\vec{c} \quad 3\hat{i} \quad \hat{j} \quad 2\hat{k}$, then find its volume. 5

Unit—II

3. (a) If $(x, y, z) \quad x^2yz \quad 4xyz^2$, then find the directional derivative of in the direction of $\vec{A} \quad 2\hat{i} \quad \hat{j} \quad 2\hat{k}$ at (1, 3, 1). 4

(b) Prove that curl$(\vec{a} \quad \vec{b}) \quad (\vec{b} \quad)\vec{a} \quad \vec{b}(\quad \vec{a}) \quad (\vec{a} \quad)\vec{b} \quad \vec{a}(\quad \vec{b})$. 6

4. (a) If $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$, then show that $r^n \quad nr^{n \; 2}\vec{r}$. 4

(b) Show that $_S \vec{F} \quad \vec{n} dS \qquad _v \quad \vec{F} dv$ where $\vec{F} \quad 4xz\hat{i} \quad y^2\hat{j} \quad yz\hat{k}$ and S is the surface of the cube bounded by $x \quad 0$, $x \quad 1$, $y \quad 0$, $y \quad 1$, $z \quad 0$, $z \quad 1$. 6

Unit—III

5. (a) Find the angle through which a set of rectangular axes must be turned without the change of origin so that the expression $7x^2 \quad 4xy \quad 3y^2$ will be transformed into the form $a x^2 \quad b y^2$. 5

(b) For what value of k will the equation $3x^2 \quad kxy \quad 3y^2 \quad 29x \quad 3y \quad 18 \quad 0$ represent a pair of straight lines? 5

6. (a) Reduce the equation $144x^2 \quad 120xy \quad 25y^2 \quad 243x \quad 448y \quad 113 \quad 0$ to the standard form and hence show that it is the equation of parabola. 6

(b) Find the vertex and length of the latus rectum of the parabola $(3x \quad 4y \quad 17)^2 \quad 35(4x \quad 3y \quad 6)$. 4

Unit—IV

7. *(a)* Find the equation of the plane which passes through the point (2, 3, 1) and is perpendicular to the line joining the points (4, 5, 2) and (2, 1, 6). 4

(b) Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes $9x\quad 7y\quad 6z\quad 48\quad 0$ and $x\quad y\quad z\quad 0$. 4

(c) Find the perpendicular distance of the points (1, 4, 2) and (5, 1, 3) from the plane $2x\quad 3y\quad z\quad 5$. 2

8. *(a)* Prove that the lines

$$\frac{x\quad 2}{3}\quad \frac{y\quad 1}{2}\quad \frac{z\quad 4}{5}$$

and $2x\quad 3y\quad z\quad 0$, $x\quad y\quad 2z\quad 4\quad 0$ are coplanar. 5

(b) Prove that the shortest distance between the lines

$$\frac{x\quad 3}{1}\quad \frac{y\quad 4}{1}\quad \frac{z\quad 1}{3} \text{ and } \frac{x\quad 1}{1}\quad \frac{y\quad 3}{3}\quad \frac{z\quad 1}{2} \text{ is } \frac{15}{\sqrt{138}}$$

and the equations of the shortest distance are $7x\quad 37y\quad 10z\quad 117\quad 0$ and $5x\quad 13y\quad 17z\quad 27\quad 0$. 5

Unit—V

9. *(a)* The plane $\frac{x}{a}\quad \frac{y}{b}\quad \frac{z}{c}\quad 1$ cuts the axes at A, B and C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC. 5

(b) Find the equation of the sphere which passes through the origin and touches the sphere $x^2\quad y^2\quad z^2\quad 56$ at the point (2, 4, 6). 5

10. *(a)* Determine the angle between the lines of intersection of the plane $x \quad 3y \quad z \quad 0$ and the quadric cone $x^2 \quad 5y^2 \quad z^2 \quad 0$. 5

(b) Find the equation of the cylinder generated by the lines parallel to the line

$$\frac{x}{1} \quad \frac{y}{2} \quad \frac{z}{1}$$

and intersecting the guiding curve $z \quad 3$ and $x^2 \quad y^2 \quad 4$. 5

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2. If $|\vec{a}|$ 4, $|\vec{b}|$ 5 and $\vec{a}$ $\vec{b}$ 0, then $\vec{a}$ $\vec{b}$ is

(a) $20\hat{n}$ ( )

(b) $9\hat{n}$ ( )

(c) $\hat{n}$ ( )

(d) 0 ( )

3. The vector $\vec{V}$ $(4x \quad 6y \quad 3z)\hat{i} \quad (2x \quad y \quad 5z)\hat{j} \quad (5x \quad 6y \quad az)\hat{k}$ is solenoidal, then the value of a is

(a) 5 ( )

(b) 8 ( )

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(d) None of the above ( )

4. Suppose V be the volume bounded by a closed surface S, $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ and $\hat{n}$ is the unit vector normal (outward) to the surface S, then

$$\vec{r} \quad \hat{n} dS$$
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7. The intercepts made on the axes by the plane $3x \quad 4y \quad 6z \quad 12 \quad 0$ are

(a) 4, 3 and 2 ( )

(b) 4, 3 and 5 ( )

(c) 5, 7 and 9 ( )

(d) None of the above ( )

8. The shortest distance between the line $\frac{x \quad 1}{4} \quad \frac{y \quad 2}{3} \quad \frac{z \quad 3}{1}$ and z-axis is

(a) $\frac{12}{5}$ ( )

(b) $\frac{11}{\sqrt{5}}$ ( )

(c) $\frac{11}{5}$ ( )

(d) $\frac{12}{7}$ ( )

9. The equation of sphere which passes through the origin and makes equal intercepts of unit length of the axes is

(a) $x^2 \quad y^2 \quad z^2 \quad 1$ ( )

(b) $(x \quad 1)^2 \quad (y \quad 1)^2 \quad (z \quad 1)^2 \quad 0$ ( )

(c) $x^2 \quad y^2 \quad z^2 \quad x \quad y \quad z \quad 1$ ( )

(d) $x^2 \quad y^2 \quad z^2 \quad x \quad y \quad z \quad 0$ ( )

10. The condition that the plane $lx \quad my \quad nz \quad 0$ touches the cone $ax^2 \quad by^2 \quad cz^2 \quad 0$ is

(a) $bcl^2 \quad cam^2 \quad abn^2 \quad 0$ ( )

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(c) $a^2l \quad b^2m \quad c^2n \quad 0$ ( )

(d) None of the above ( )

SECTION—B

(*Marks* : 15)

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State *True* or *False* by putting a Tick (✓) mark in the brackets provided and give a brief justification :

1. If $\vec{a} \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k}$, $\vec{b} \quad \hat{i} \quad 3\hat{j} \quad 5\hat{k}$ and $\vec{c} \quad \hat{i} \quad \hat{j} \quad 6\hat{k}$, then the value of $\vec{a} \quad (\vec{b} \quad \vec{c})$ is 5.

True ( ) *False* ( )

Justification :

2. The value of $\frac{1}{r}$, where $\vec{r}\quad x\hat{i}\quad y\hat{j}\quad z\hat{k}$ is $\frac{\vec{r}}{r^3}$.

True ( ) *False* ( )

Justification :

3. The equation of the diameter of the conic $4x^2\quad 6xy\quad 5y^2\quad 1$ conjugate to the diameter $y\quad 2x\quad 0$ is $10y\quad 7x\quad 0$.

True ( ) *False* ( )

Justification :

4. The equation of the plane through the line $x\quad y\quad z\quad 3\quad 0,\ 2x\quad y\quad 3z\quad 1\quad 0$ and parallel to the line $\frac{x}{1}\quad \frac{y}{2}\quad \frac{z}{3}$ is $x\quad 5y\quad 3z\quad 7\quad 0$.

True ( ) *False* ( )

Justification :

5. The equation of the orthogonal projection of the curve $2x\quad y\quad z\quad 3$, $x^2\quad 2y^2\quad 3z^2\quad 1$ on the z-plane is

$$z\quad 0,\ 13x^2\quad 5y^2\quad 36x\quad 18y\quad 12xy\quad 26\quad 0$$

True ( ) *False* ( )

Justification :

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

Unit—I

1. *(a)* Find a unit vector perpendicular to the plane of $\vec{A}\quad 2\hat{i}\quad 6\hat{j}\quad 3\hat{k}$ and $\vec{B}\quad 4\hat{i}\quad 3\hat{j}\quad \hat{k}$. 3

(b) If ABC be a triangle, then prove that

$$\cos A\quad \frac{b^2\quad c^2\quad a^2}{2bc}$$ 3

(c) Find the set of vectors reciprocal to the set $\hat{i} \quad 2\hat{j} \quad 3\hat{k}$, $5\hat{i} \quad \hat{j} \quad \hat{k}$ and $\hat{i} \quad \hat{j} \quad \hat{k}$. 4

2. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then prove that $[\vec{a} \quad \vec{b} \;\; \vec{b} \quad \vec{c} \;\; \vec{c} \quad \vec{a}] \quad [\vec{a} \; \vec{b} \; \vec{c}]$. 5

(b) If three concurrent edges of a parallelepiped is given by $\vec{a} \quad 2\hat{i} \quad 3\hat{j} \quad 4\hat{k}$, $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$ and $\vec{c} \quad 3\hat{i} \quad \hat{j} \quad 2\hat{k}$, then find its volume. 5

Unit—II

3. (a) If $\;(x, y, z) \quad x^2yz \quad 4xyz^2$, then find the directional derivative of $\;$ in the direction of $\vec{A} \quad 2\hat{i} \quad \hat{j} \quad 2\hat{k}$ at (1, 3, 1). 4

(b) Prove that $\text{curl}(\vec{a} \quad \vec{b}) \quad (\vec{b} \quad)\vec{a} \quad \vec{b}(\quad \vec{a}) \quad (\vec{a} \quad)\vec{b} \quad \vec{a}(\quad \vec{b})$. 6

4. (a) If $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$, then show that $\quad r^n \quad nr^{n \; 2}\vec{r}$. 4

(b) Show that $\quad_S \vec{F} \quad \vec{n}dS \qquad_v \quad \vec{F}dv$ where $\vec{F} \quad 4xz\hat{i} \quad y^2\hat{j} \quad yz\hat{k}$ and S is the surface of the cube bounded by $x \quad 0$, $x \quad 1$, $y \quad 0$, $y \quad 1$, $z \quad 0$, $z \quad 1$. 6

Unit—III

5. (a) Find the angle through which a set of rectangular axes must be turned without the change of origin so that the expression $7x^2 \quad 4xy \quad 3y^2$ will be transformed into the form $a\,x^2 \quad b\,y^2$. 5

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Unit—IV

7. *(a)* Find the equation of the plane which passes through the point (2, 3, 1) and is perpendicular to the line joining the points (4, 5, 2) and (2, 1, 6). 4

(b) Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes $9x \quad 7y \quad 6z \quad 48 \quad 0$ and $x \quad y \quad z \quad 0$. 4

(c) Find the perpendicular distance of the points (1, 4, 2) and (5, 1, 3) from the plane $2x \quad 3y \quad z \quad 5$. 2

8. *(a)* Prove that the lines

$$\frac{x \quad 2}{3} \quad \frac{y \quad 1}{2} \quad \frac{z \quad 4}{5}$$

and $2x \quad 3y \quad z \quad 0$, $x \quad y \quad 2z \quad 4 \quad 0$ are coplanar. 5

(b) Prove that the shortest distance between the lines

$$\frac{x \quad 3}{1} \quad \frac{y \quad 4}{1} \quad \frac{z \quad 1}{3} \text{ and } \frac{x \quad 1}{1} \quad \frac{y \quad 3}{3} \quad \frac{z \quad 1}{2} \text{ is } \frac{15}{\sqrt{138}}$$

and the equations of the shortest distance are $7x \quad 37y \quad 10z \quad 117 \quad 0$ and $5x \quad 13y \quad 17z \quad 27 \quad 0$. 5

Unit—V

9. *(a)* The plane $\frac{x}{a} \quad \frac{y}{b} \quad \frac{z}{c} \quad 1$ cuts the axes at A, B and C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC. 5

(b) Find the equation of the sphere which passes through the origin and touches the sphere $x^2 \quad y^2 \quad z^2 \quad 56$ at the point (2, 4, 6). 5

10. *(a)* Determine the angle between the lines of intersection of the plane $x \quad 3y \quad z \quad 0$ and the quadric cone $x^2 \quad 5y^2 \quad z^2 \quad 0$. 5

(b) Find the equation of the cylinder generated by the lines parallel to the line

$$\frac{x}{1} \quad \frac{y}{2} \quad \frac{z}{1}$$

and intersecting the guiding curve $z \quad 3$ and $x^2 \quad y^2 \quad 4$. 5

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(CBCS)

(4th Semester)

MATHEMATICS

FOURTH PAPER

(Vector Calculus and Solid Geometry)

Full Marks : 75

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(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Tick (✓) the correct answer in the brackets provided :

1. The component of $\vec{a} \quad 2\hat{i} \quad \hat{j} \quad \hat{k}$ on $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$ is

(a) $\frac{1}{6}(\hat{i} \quad 2\hat{j} \quad \hat{k})$ ( )

(b) $\frac{5}{6}(\hat{i} \quad 2\hat{j} \quad \hat{k})$ ( )

(c) $\frac{1}{6}(2\hat{i} \quad \hat{j} \quad \hat{k})$ ( )

(d) $\frac{5}{6}(2\hat{i} \quad \hat{j} \quad \hat{k})$ ( )

2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be any three vectors, then $\vec{a} \quad (\vec{b} \quad \vec{c}) \quad \vec{b} \quad (\vec{c} \quad \vec{a}) \quad \vec{c} \quad (\vec{a} \quad \vec{b})$ is

(a) $\vec{0}$ ( )

(b) $\vec{a}$ ( )

(c) $\vec{b}$ ( )

(d) $\vec{c}$ ( )

3. If is a scalar point function, then grad is

(a) both solenoidal and irrotational ( )

(b) solenoidal ( )

(c) irrotational ( )

(d) neither solenoidal nor irrotational ( )

4. If $^{p_2}_{p_1} \vec{F} \quad d\vec{r}$ is independent of the path joining the two points a and b in a given region, then for all closed paths in the region, $\circ \vec{F} \quad d\vec{r}$ is

(a) p_2 ( )

(b) p_1 ( )

(c) 0 ( )

(d) None of the above ( )

5. The equation $ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$ represents a hyperbola, if

(a) $ab \quad h^2 \quad 0$ ( )

(b) $ab \quad h^2 \quad 0$ ( )

(c) $ab \quad h^2 \quad 0$ ( )

(d) $a \quad b$ and $h \quad 0$ ( )

6. The centre of the conic given by the equation $3x^2 \quad 8xy \quad 7y^2 \quad 4x \quad 2y \quad 7 \quad 0$ is

(a) (2, –1) ( )

(b) (1, –2) ( )

(c) (2, 1) ( )

(d) (1, 2) ( )

7. The intercepts on z-axis by the plane $x \quad y \quad 2z \quad 2$ is

(a) 1 ( )

(b) 2 ( )

(c) 3 ( )

(d) 4 ( )

8. The angle between the planes $x \quad y \quad z \quad 1$ and $x \quad y \quad 2$ is

(a) 0 ( )

(b) $\frac{}{2}$ ( )

(c) $\frac{}{3}$ ( )

(d) $\frac{}{4}$ ( )

9. The equation of sphere which passes through the origin and makes equal intercepts of unit length of the axes is

(a) $x^2 \quad y^2 \quad z^2 \quad 1$ ( )

(b) $x^2 \quad y^2 \quad z^2 \quad x \quad y \quad z \quad 0$ ( )

(c) $x^2 \quad y^2 \quad z^2 \quad x \quad y \quad z \quad 0$ ( )

(d) $(x \quad 1)^2 \quad (y \quad 1)^2 \quad (z \quad 1)^2 \quad 0$ ( )

10. The reciprocal cone of the cone $ax^2 \quad by^2 \quad cz^2 \quad 0$ is

(a) $x^2 \quad y^2 \quad z^2 \quad 0$ ( )

(b) $bcx^2 \quad cay^2 \quad abz^2 \quad 0$ ( )

(c) $x^2 \quad y^2 \quad z^2 \quad a^2 \quad b^2 \quad c^2$ ( )

(d) Does not exist ( )

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. (a) For any vector $\vec{a}$, prove that $\hat{i} \quad (\vec{a} \quad \hat{i}) \quad \hat{j} \quad (\vec{a} \quad \hat{j}) \quad \hat{k} \quad (\vec{a} \quad \hat{k}) \quad 2\vec{a}$.

OR

(b) A particle moves along a curve whose parametric equations are $x \quad e^{\quad t}$, $y \quad 2\cos 3t$, $z \quad 2\sin 3t$, where t is the time. Determine its velocity and acceleration at any time.

2. (a) Prove that $\quad (\quad \vec{F}) \quad 0$.

OR

(b) Find div$\vec{f}$ at $(1, \quad 1, 1)$, if $\vec{f} \quad x^2 z\hat{i} \quad 2y^3 z^2 \hat{j} \quad xy^2 z\hat{k}$.

3. *(a)* Prove that the diameter of the conic $15x^2 \quad 20xy \quad 16y^2 \quad 1$ conjugate to the diameter $y \quad 2x \quad 0$ is $5x \quad 6y$.

OR

(b) Show that the equation of the asymptotes of the hyperbola $2x^2 \quad 5xy \quad 3y^2 \quad 5x \quad 3y \quad 21 \quad 0$ is $2x^2 \quad 5xy \quad 3y^2 \quad 5x \quad 3y \quad \frac{18}{49} \quad 0$.

4. *(a)* Prove that the lines

$$\frac{x \quad 3}{2} \quad \frac{y \quad 5}{3} \quad \frac{z \quad 7}{3}, \frac{x \quad 1}{4} \quad \frac{y \quad 1}{5} \quad \frac{z \quad 1}{1}$$

are coplanar.

OR

(b) Prove that the length of the perpendicular drawn from the point (x_1, y_1, z_1) to the plane $ax \quad by \quad cz \quad d \quad 0$ is

$$\frac{|ax_1 \quad by_1 \quad cz_1 \quad d|}{\sqrt{a^2 \quad b^2 \quad c^2}}$$

5. *(a)* Show that the general equation of a cone which touches the three co-ordinate planes is $\sqrt{fx} \quad \sqrt{gy} \quad \sqrt{hz} \quad 0$.

OR

(b) Find the equation of the sphere through the circle $x^2 \quad y^2 \quad z^2 \quad 9$, $2x \quad 3y \quad 4z \quad 5$ and the point (1, 2, 3).

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Prove that for a triangle ABC, $\frac{\sin A}{a} \quad \frac{\sin B}{b} \quad \frac{\sin C}{c}$, where $AB \quad c$, $BC \quad a$ and $CA \quad b$. 5

(b) A particle moves along the curve $x \quad 2t^2$, $y \quad t^2 \quad 4t$, $z \quad 3t \quad 5$, where t is the time. Find the components of its velocity and acceleration at time $t \quad 1$ in the direction $\hat{i} \quad 3\hat{j} \quad 2\hat{k}$. 5

2. *(a)* Prove that a vector function $\vec{f}(t)$ will be of constant magnitude, if and only if $\vec{f} \quad \frac{d\vec{f}}{dt} \quad 0$. 5

(b) Find the value of so that the four points with position vectors $A(\quad 6\hat{i} \quad 3\hat{j} \quad 2\hat{k})$, $B(3\hat{i} \quad \hat{j} \quad 4\hat{k})$, $C(5\hat{i} \quad 7\hat{j} \quad 3\hat{k})$ and $D(\quad 13\hat{i} \quad 17\hat{j} \quad 2\hat{k})$ are coplanar. 5

UNIT—II

3. *(a)* Let $(x, y, z) \quad x^3 \quad y^3 \quad z^2$. Find the directional derivative of at $(1, \quad 1, 2)$ in the direction of the vector $\hat{i} \quad 2\hat{j} \quad \hat{k}$. 5

(b) Suppose $\vec{A} \quad 0$. Evaluate $(\vec{A} \quad \vec{r}) \quad 0$, where $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ and $\vec{A} \quad A_1\hat{i} \quad A_2\hat{j} \quad A_3\hat{k}$. 5

4. *(a)* Evaluate $\underset{S}{} \vec{A} \ \vec{n} dS$, where $\vec{A} \quad y\hat{i} \quad 2x\hat{j} \quad z\hat{k}$ and S is the surface of the plane $2x \quad y \quad 6$ in the first octant cut off by the plane $z \quad 4$. 5

(b) Find the work done in moving a particle in the force field $\vec{F} \quad 3x^2\hat{i} \quad (2xz \quad y)\hat{j} \quad z\hat{k}$ along—

(i) a straight line from (0, 0, 0) to (2, 1, 3);

(ii) the curve defined by $x^2 \quad 4y$, $3x^3 \quad 8z$ from $x \quad 0$ to $x \quad 2$. 5

UNIT—III

5. *(a)* If, by a rotation of the rectangular axes about the origin, the expression $ax^2 \quad 2hxy \quad by^2$ changes to $a_1x_1^2 \quad 2h_1x_1y_1 \quad b_1y_1^2$, then show that $a \quad b \quad a_1 \quad b_1$ and $ab \quad h^2 \quad a_1b_1 \quad h_1^2$. 5

(b) Find the equations of the parabolas passing through the points of intersection of $x^2 \quad 6xy \quad y^2 \quad 2x \quad 3y \quad 5 \quad 0$ and $2x^2 \quad 8xy \quad 3y^2 \quad 2y \quad 1 \quad 0$. 5

6. *(a)* Reduce the equation of $144x^2 \quad 120xy \quad 25y^2 \quad 243x \quad 448y \quad 113 \quad 0$ to the standard form and hence show that it is the equation of a parabola. 5

(b) Prove that the straight lines represented by the equation $ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$ will be equidistant from the origin, if $f^4 \quad g^4 \quad c \quad (bf^2 \quad ag^2)$. 5

UNIT—IV

7. *(a)* A variable plane is at a constant distance $3p$ from the origin and meet the axes in *A*, *B* and *C*. Show that the locus of the centroid of the triangle *ABC* is $x^{\ 2} \quad y^{\ 2} \quad z^{\ 2} \quad p^{\ 2}$. 5

(b) Show that the lines $\frac{x \quad a \quad d}{\quad} \quad \frac{y \quad a}{\quad} \quad \frac{z \quad a \quad d}{\quad}$ and $\frac{x \quad b \quad c}{\quad} \quad \frac{y \quad b}{\quad} \quad \frac{z \quad b \quad c}{\quad}$ are coplanar. 5

8. *(a)* Find the equation of the plane passing through the line of intersection of the planes $x \quad 2y \quad z \quad 1$ and $2x \quad y \quad z \quad 8$, and parallel to the line with direction ratios 1, 2, 1. Find also the perpendicular distance of (1, 1, 1) from this plane. 5

(b) Show that the lines $\frac{x \quad 1}{2} \quad \frac{y \quad 2}{3} \quad \frac{z \quad 3}{4}$ and $\frac{x \quad 4}{5} \quad \frac{y \quad 1}{2} \quad z$ intersect each other. Find the point of their intersection. 5

UNIT—V

9. *(a)* Show that the condition for the plane $lx \quad my \quad nz \quad p$ to be a tangent plane to $x^2 \quad y^2 \quad z^2 \quad a^2$ is $a^2(l^2 \quad m^2 \quad n^2) \quad p^2$. 5

(b) Find the radius of the circle, where the plane $x \quad 2y \quad 2z \quad 3$ intersects the sphere $x^2 \quad y^2 \quad z^2 \quad 8x \quad 4y \quad 8z \quad 45$. 5

10. *(a)* Find the equation of a right circular cylinder of radius 5, whose axis passes through (1, 2, 3) and is parallel to $\frac{x \quad 4}{2} \quad \frac{y \quad 3}{1} \quad \frac{z \quad 2}{2}$. 5

(b) Find the equation of the cone whose vertex is (, ,) and base is $ax^2 \quad by^2 \quad 1, \; z \quad 0$. 5

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