2 0 1 8

(CBCS)

(3rd Semester)

MATHEMATICS

THIRD PAPER

(Differential Equations)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick ☑ the correct answer in the box provided : 1×10=10

1. The differential equation of all circles passing through the origin and having circles on the axis is

(a) $x^2 \quad y^2 \quad 2xy\frac{dy}{dx}$ ☐

(b) $x^2 \quad y^2 \quad 2xy\frac{dy}{dx}$ ☐

(c) $x^2 \quad y^2 \quad 2xy\frac{dy}{dx}$ ☐

(d) $x^2 \quad y^2 \quad 2xy\frac{dy}{dx}$ ☐

2. The general solution of the differential equation, $(x \quad 1)y\,dx \quad x(1 \quad y)dy$ is

(a) $x \quad y \quad \log \dfrac{x}{y} \quad c$ ☐

(b) $x \quad y \quad \log \dfrac{x}{y} \quad c$ ☐

(c) $x \quad y \quad \log(xy) \quad c$ ☐

(d) $x \quad y \quad \log \; xy \quad c$ ☐

3. The complementary function of $(D^2 \quad 2D \quad 5)y \quad \cos 3x$ is

(a) $c_1 e^{\ 2x} \quad c_2 e^{3x}$ ☐

(b) $(c_1 \quad c_2 x)e^{x}$ ☐

(c) $e^{x}(c_1 \cos 2x \quad c_2 \sin 2x)$ ☐

(d) $e^{2x}(c_1 \cos x \quad c_2 \sin x)$ ☐

4. The particular integral of $(D^2 \quad 4D \quad 4)y \quad e^{\ 2x}$ is

(a) $\dfrac{xe^{\ 2x}}{2}$ ☐

(b) $\dfrac{e^{\ 2x}}{2}$ ☐

(c) $\dfrac{x^2 e^{\ 2x}}{3}$ ☐

(d) $\dfrac{x^2 e^{\ 2x}}{5}$ ☐

5. The orthogonal trajectories to the family of parabolas $y \quad cx^2$ are

(a) parabolas ☐

(b) hyperbolas ☐

(c) ellipses ☐

(d) None of the above ☐

6. The complete solution of $(D^3 \quad 6D^2 \quad 11D \quad 6)y \quad 0$, where $D \quad \frac{d}{dx}$, is

(a) $y \quad c_1 e^{x} \quad c_2 e^{2x} \quad c_3 e^{3x}$ ☐

(b) $y \quad c_1 e^{\ x} \quad c_2 e^{\ 2x} \quad c_3 e^{\ 3x}$ ☐

(c) $y \quad c_1 e^{x} \quad c_2 e^{3x} \quad c_3 e^{5x}$ ☐

(d) $y \quad c_1 e^{\ x} \quad c_2 e^{\ 3x} \quad c_3 e^{\ 5x}$ ☐

7. The transformed equation of $x^6 \frac{d^2y}{dx^2} \quad 3x^5 \frac{dy}{dx} \quad ay \quad \frac{1}{x^2}$ is

(a) $\frac{d^2y}{dz^2} \quad 2y \quad \frac{2z}{a}$ ☐

(b) $\frac{d^2y}{dz^2} \quad 2y \quad \frac{2z}{a^3}$ ☐

(c) $\frac{d^2y}{dz^2} \quad y \quad \frac{2z}{a}$ ☐

(d) $\frac{d^2y}{dz^2} \quad y \quad \frac{2z}{a^3}$ ☐

8. The particular integral of $(1 \quad x^2)\frac{d^2y}{dx^2} \quad (1 \quad x)\frac{dy}{dx} \quad y \quad 4 \cos \log (1 \quad x)$ is

(a) $2 \log (1 \quad x) \sin \log (1 \quad x)$ ☐

(b) $2 \log (1 \quad x) \cos \log (1 \quad x)$ ☐

(c) $\log (1 \quad x) \sin \log (1 \quad x)$ ☐

(d) $\log (1 \quad x) \cos \log (1 \quad x)$ ☐

9. The partial differential equation of $z \quad f(x \quad iy) \quad F(x \quad iy)$ by eliminating f and F is

(a) $\frac{{}^2 z}{x^2} \quad \frac{{}^2 z}{y^2} \quad 0$ ☐

(b) $\frac{{}^2 z}{x^2} \quad \frac{{}^2 z}{y^2} \quad 0$ ☐

(c) $\frac{{}^2 z}{x^2} \quad \frac{{}^2 z}{y^2} \quad 0$ ☐

(d) None of the above ☐

10. The general solution of partial differential equation $p \quad q \quad 1$ is

(a) $(x \quad z, \; y \quad z) \quad 0$ ☐

(b) $(x \quad z, \; y \quad z) \quad 0$ ☐

(c) $(x \quad z, \; y \quad z) \quad 0$ ☐

(d) $(x \quad z, \; y \quad z) \quad 0$ ☐

SECTION—B

(*Marks* : 15)

Each question carries *3* marks

1. Solve :

$$x\frac{dy}{dx} \quad y \quad x \tan \frac{y}{x}$$

OR

Test the exactness of the differential equation

$$1 \quad 3e^{\frac{x}{y}} \; dx \quad 3e^{\frac{x}{y}}(1 \quad xy \quad ^{1})\,dy \quad 0$$

2. Solve the differential equation

$$x\frac{dy}{dx} \quad y \tan x \quad \sec x$$

OR

Solve the differential equation $\frac{d^2y}{dx^2} \quad 2\frac{dy}{dx} \quad 15 \quad e^{3x}$, where $\frac{d}{dx} \quad D$.

3. Solve :

$$x^2 \frac{d^2y}{dx^2} \quad 4x\frac{dy}{dx} \quad 6y \quad x$$

OR

Solve :

$$(1 \quad x^2)\frac{d^2y}{dx^2} \quad 3x\frac{dy}{dx} \quad y \quad 0$$

4. Find the general and singular solution of $y \quad px \quad \frac{a}{p}, \; p \quad \frac{dy}{dx}$.

OR

Find the orthogonal trajectories of the system of curves $y^2 \quad 4a(x \quad a)$, a being the parameter.

5. Find the surface satisfying the partial differential equation $(x \quad y)y^2 p \quad (y \quad x)x^2 q \quad (x^2 \quad y^2)z$, which passes the curve $xz \quad a^3$, $y \quad 0$.

OR

Find the complete integral of $p^3 \quad q^3 \quad 27z$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Reduce the equation $(x^3y^2 \quad xy)\,dx \quad dy$ to a linear differential equation and solve it. 5

(b) Solve : 5

$$\frac{dy}{dx} \quad \frac{x \quad y \quad 2}{x \quad 2y \quad 3} \quad 0$$

2. *(a)* Reduce the equation $x\,dy \quad y\,dx \qquad xy^2\,dx$ to exact form and solve it. 5

(b) Reduce the following equation $\dfrac{dy}{dx} \quad \dfrac{\sin(2y)}{x} \quad x^3\cos^2 y$ to linear differential equation and solve it. 5

UNIT—II

3. *(a)* Solve : 5

$$\frac{d^2y}{dx^2} \quad x\frac{dy}{dx} \quad 4y \quad e^x \cos x$$

(b) Solve : 5

$$\frac{d^3y}{dx^3} \quad 3\frac{d^2y}{dx^2} \quad 4\frac{dy}{dx} \quad 2y \quad e^x$$

4. *(a)* Solve : 5

$$(D^2 \quad 9)y \quad e^{2x} \quad \sin 3x$$

(b) Solve : 5

$$(D^2 \quad 7D \quad 12)y \quad x^3$$

UNIT—III

5. *(a)* Find the orthogonal trajectories of the family of curves given by $r \quad \dfrac{2a}{1 \quad \cos}$. 5

(b) By substituting $x^2 \quad u$ and $y^2 \quad v$, reduce $x^2(y \quad px) \quad yp^2$ into Clairaut's form and find the singular solution. 5

6. *(a)* Find the orthogonal trajectory of $r \quad a(1 \quad \cos \ )$. 5

(b) Solve : 2½+2½=5

(i) $y \quad px \quad p^2x^4$

(ii) $xyp^2 \quad (x^2 \quad y^2)p \quad xy \quad 0$

UNIT—IV

7. *(a)* By using the method of variation of parameters, solve the equation

$$y_2 \quad 3y_1 \quad 2y \quad \frac{e^x}{1 \quad e^x}$$ 6

(b) Solve : 4

$$x\frac{d^2y}{dx^2} \quad \frac{dy}{dx} \quad 4x^3y \quad x^5$$

8. *(a)* Solve the simultaneous differential equations $\frac{dy}{dx} \quad 3z \quad y$ and $\frac{dz}{dx} \quad 4z \quad 2y$. 5

(b) Solve : 5

$$\frac{d^2y}{dx^2} \quad 2\tan x\frac{dy}{dx} \quad y \quad 0$$

UNIT—V

9. *(a)* Find the equation of integral surface of the differential equations

$$2y(z \quad 3)p \quad (2x \quad z)q \quad y(2x \quad 3)$$ 5

(b) Apply Charpit's method and solve

$$2xz \quad px^2 \quad 2xyq \quad pq \quad z$$ 5

10. *(a)* Find the equation of integral surface of the differential equation $(x \quad y)p \quad (y \quad x \quad z)q \quad z$ through the circle $z \quad 1$, $x^2 \quad y^2 \quad 1$. 5

(b) Solve : 5

$$(p^2 \quad q^2)y \quad qz$$

★ ★ ★

MATH/III/03 **Student's Copy**

2 0 1 8

(Pre-CBCS)

(3rd Semester)

MATHEMATICS

THIRD PAPER

(Differential Equations)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick ☑ the correct answer in the box provided : 1×10=10

1. If $y \quad \frac{2}{x \quad 2}$, then the corresponding differential equation satisfying to it, is

(a) $x \quad yy \quad x^2$ in $(\quad , \quad 2)$ ☐

(b) $xy \quad y \quad y^2$ in $(\quad , \quad 2) \quad (\quad 2, \quad)$ ☐

(c) $x \quad yy \quad x^2$ in $(2, \quad)$ ☐

(d) $xy \quad y \quad y^2$ in $(2, \quad)$ ☐

2. The differential equation of the family of parabolas $y^2 \quad 4ax$ is

(a) $y \quad 2x\dfrac{dy}{dx}$ ☐

(b) $x \quad 2y\dfrac{dy}{dx}$ ☐

(c) $y \quad 4x\dfrac{dy}{dx}$ ☐

(d) $x \quad 4y\dfrac{dy}{dx}$ ☐

3. The general solution of the differential equation $(D^2 \quad 1)^2 y \quad 0$, where $D \quad \dfrac{d}{dx}$ is

(a) $y \quad C_1 \cos x \quad C_2 \sin x$ ☐

(b) $y \quad (C_1 \quad C_2)\cos x \quad (C_1 \quad C_2)\sin x$ ☐

(c) $y \quad C_1 x \cos x \quad C_2 x^2 \sin x$ ☐

(d) $y \quad (C_1 \quad C_2 x)\cos x \quad (C_3 \quad C_4 x)\sin x$ ☐

4. The particular integral (PI) of the differential equation $(D^4 \quad D^2)y \quad 2$, where $D \quad \dfrac{d}{dx}$ is

(a) x ☐

(b) x^2 ☐

(c) $\quad x^2$ ☐

(d) $\dfrac{x}{2}$ ☐

5. The general solution of the differential equation $p \quad \log(px \quad y)$, where $p \quad \dfrac{dy}{dx}$ is

(a) $y \quad Cx \quad \log C$ ☐

(b) $y \quad x \quad C$ ☐

(c) $y \quad Cx \quad e^C$ ☐

(d) $y \quad C(x \quad 1)$ ☐

6. The singular solution of $xp^2 \quad (y \quad x)p \quad y \quad 1$, where $p \quad \frac{dy}{dx}$ is

(a) $x \quad 0$ ☐

(b) $y \quad 0$ ☐

(c) $(x \quad y)^2 \quad 4y \quad 0$ ☐

(d) $(x \quad y)^2 \quad 4x \quad 0$ ☐

7. $y \quad x^2$ is a solution of $y \quad P(x)y \quad Q(x)y \quad 0$ if

(a) $1 \quad xP \quad x^2Q \quad 0$ ☐

(b) $1 \quad P \quad Q \quad 0$ ☐

(c) $2 \quad 2xP \quad x^2Q \quad 0$ ☐

(d) None of the above ☐

8. If $zy\,dx \quad zx\,dy \quad y^2dz$ is integrable, then the general solution is

(a) $z \quad Ce^{\frac{y}{x}}$ ☐

(b) $z \quad Ce^{\frac{x}{y}}$ ☐

(c) $z \quad Ce^{x \quad y}$ ☐

(d) $z \quad Ce^{x \quad y}$ ☐

9. The differential equation of all surfaces of revolution having z-axis as the axis of rotation is

(a) $y\frac{z}{x} \quad x\frac{z}{y}$ ☐

(b) $x\frac{z}{x} \quad y\frac{z}{y}$ ☐

(c) $\frac{z}{x} \quad \frac{z}{y}$ ☐

(d) $\frac{z}{x} \quad \frac{z}{y} \quad 0$ ☐

10. The differential equations $\frac{z}{x} \quad 5x \quad 7y, \frac{z}{y} \quad 6x \quad 8y$ are

(a) compatible and have common solutions ☐

(b) not compatible and have no solutions ☐

(c) not compatible and have solutions ☐

(d) compatible and have no solutions ☐

SECTION—B

(*Marks* : 15)

Each question carries *3* marks

1. Solve the differential equation

$$\frac{dy}{dx} \quad y \cot x \quad 2 \cos x$$

2. Solve the differential equation $(D^2 \quad 1)y \quad \operatorname{cosec} x$ where $D \quad \frac{d}{dx}$.

3. Assuming that a spherical rain drop evaporates at a rate proportional to its surface area. If its radius originally is 3 mm, and one hour later it has been reduced to 2 mm, then find an expression for the radius of the rain drop at any time.

4. Solve the simultaneous equations :

$$\frac{dy}{dx} \quad y \quad z \quad e^x$$

$$\frac{dz}{dx} \quad z \quad y \quad e^x$$

5. Solve the partial differential equation $xzp \quad yzq \quad xy$, where $p \quad \frac{z}{x}, q \quad \frac{z}{y}$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Solve the differential equation

$$\frac{dy}{dx} \quad \sec(x \quad y)$$ 5

(b) Solve the differential equation

$$\frac{dy}{dx} \quad \frac{2y}{x} \quad \sin x$$ 5

2. *(a)* Solve the differential equation

$$(xy^2 \quad x^2)dx \quad (3x^2y^2 \quad x^2y \quad 2x^3 \quad y^2)dy \quad 0$$ 5

(b) Solve the differential equation

$$\frac{dy}{dx} \quad \frac{x \quad y \quad 4}{x \quad y \quad 6}$$ 5

UNIT—II

3. *(a)* Solve the differential equation

$$(D^2 \quad 1)y \quad \sin x \ \sin 2x$$

where $D \quad \frac{d}{dx}$. 5

(b) Solve the differential equation

$$(D^3 \quad D^2 \quad 3D \quad 5)y \quad e^x \cos x$$

where $D \quad \frac{d}{dx}$. 5

4. *(a)* Solve the differential equation

$$(D^2 \quad 4)y \quad \tan 2x$$

where $D \quad \frac{d}{dx}$. 5

(b) Solve the differential equation

$$(xD^3 \quad D^2)y \quad \frac{1}{x}$$

where $D \quad \frac{d}{dx}$. 5

UNIT—III

5. *(a)* Solve the differential equation

$$y \quad 2px \quad y^2 p^3$$

where $p \quad \frac{dy}{dx}$. 5

(b) Solve the differential equation

$$y \quad 2px \quad p^2$$

where $p \quad \frac{dy}{dx}$. 5

6. *(a)* Reduce the differential equation

$$(px \quad y)(x \quad py) \quad 2p$$

to Clairaut's form by substitution $x^2 \quad u$ and $y^2 \quad v$, and find its complete primitive and singular solutions. 5

(b) Find the orthogonal trajectories of

$$x^2 \quad y^2 \quad 2ax$$

a being a parameter. 5

UNIT—IV

7. *(a)* Solve the differential equation

$$x^2 \frac{d^2y}{dx^2} \quad x\frac{dy}{dx} \quad y \quad x^2 e^x$$

5

(b) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} \quad y \quad \frac{2}{1 \quad e^x}$$ 5

8. (a) Solve the differential equation

$$y \quad \frac{2y}{x} \quad 1 \quad \frac{2}{x^2} \; y \quad xe^x$$

by changing the dependent variable. 5

(b) Verify that

$$(x^2z \quad y^3)dx \quad 3xy^2dy \quad x^3dz \quad 0$$

is integrable and solve it. 5

UNIT—V

9. (a) Solve the differential equation

$$x(y^2 \quad z^2)p \quad y(z^2 \quad x^2)q \quad z(x^2 \quad y^2)$$ 5

(b) Solve the differential equation

$$px(z \quad 2y^2) \quad (z \quad qy)(z \quad y^2 \quad 2x^3)$$ 5

10. (a) Find the integral surface of the partial differential equation

$$(x \quad y)y^2p \quad (y \quad x)x^3q \quad (x^2 \quad y^2)z$$

passing through the curve $xz \quad a^3$, $y \quad 0$. 5

(b) Find a complete, singular and general integral of

$$(p^2 \quad q^2)y \quad qz$$ 5

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