2 0 1 8

(Pre-CBCS)

(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The matrix

$$A \quad \begin{matrix} 0 & 1 & 8 \\ 1 & 0 & 2 \\ 8 & 2 & 0 \end{matrix}$$

is

(a) Hermitian but not symmetric ( )

(b) symmetric but not Hermitian ( )

(c) skew-symmetric but not Hermitian ( )

(d) symmetric and Hermitian ( )

2. The rank of the matrix

$$A \quad \begin{matrix} 1 & 2 & 8 \\ 2 & 3 & 4 \\ 10 & 2 & 2 \end{matrix}$$

is

(a) 3 ( )

(b) 1 ( )

(c) 2 ( )

(d) 0 ( )

3. The number of binary compositions on a finite set A having n elements is

(a) $\frac{n^2 \quad n}{2}$ ( )

(b) n^{n^2} ( )

(c) 2^{n^2} ( )

(d) n^n ( )

4. The multiplicative group $G \quad \{1, \quad , \quad ^2\}$ is cyclic. Then the generators are

(a) 1 and ( )

(b) 1 and ^{2} ( )

(c) 1 and 2 ( )

(d) and ^{2} ( )

5. On dividing 11^7 by 18, the remainder is

(a) 12 ( )

(b) 11 ( )

(c) 13 ( )

(d) 18 ( )

6. If f is a homomorphism of G into G, then the set K of all those elements of G which are mapped by f onto the identity element of G is called

(a) kernel of the homomorphism f ( )

(b) homomorphism f ( )

(c) kernel of the isomorphism f ( )

(d) isomorphism f ( )

7. The expression $x^5 \quad 61x \quad p$ is divided by $(x \quad 1)$. Then the value of p is

(a) 62 ( )

(b) 60 ( )

(c) –60 ( )

(d) 6 ( )

8. If $f(x)$ and $g(x)$ are non-zero polynomials in $F[x]$, then $f(x) \quad g(x)$ is non-zero and $\deg(f(x) \quad g(x))$ is

(a) $\deg(f(x)) \quad \deg(g(x))$ ( )

(b) $\max\{\deg(f(x)), \deg(g(x))\}$ ( )

(c) $\deg\{f(x)\} \quad \deg\{g(x)\}$ ( )

(d) $\min\{\deg(f(x)), \deg(g(x))\}$ ( )

9. The equation $4x^3 \quad 13x^2 \quad 31x \quad 41 \quad 0$ has

(a) three positive roots ( )

(b) one positive root which lies between 0 and 1 ( )

(c) no positive root ( )

(d) only one positive root which lies between 1 and 2 ( )

10. One root of the equation $2x^3 \quad 21x^2 \quad 42x \quad 16 \quad 0$, whose roots are known to be in GP is

(a) 1 ( )

(b) 2 ( )

(c) −2 ( )

(d) −1 ( )

SECTION—B

(*Marks* : 15)

State True or False of the following with a brief justification : 3×5=15

1. If A be a skew-Hermitian matrix, then iA is also skew-Hermitian matrix, where $i \quad \sqrt{\quad 1}$.

(True / False)

Justification :

2. Every cyclic group is an Abelian group.

(True / False)

Justification :

3. Every homomorphic image of an Abelian group is Abelian.

(True / False)

Justification :

4. $(2x \quad 1)$ is a factor of $4x^5 \quad 3x^3 \quad 6x^2 \quad 5$.

(True / False)

Justification :

5. The De Moivre's form of complex number $3 \quad 4i$ is $4(\cos \quad i\sin \quad)$, where $\tan^{\ 1} \frac{4}{3}$.

(True / False)

Justification :

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* If A and B are invertible matrices of the same order, show that AB is invertible and $(AB)^{1} \quad B^{1}A^{1}$. 5

(b) Find the inverse of

$$A \quad \begin{matrix} 3 & 1 & 4 \\ 0 & 2 & 1 \\ 1 & 1 & 2 \end{matrix}$$

by elementary operation. 5

2. *(a)* Using Cayley-Hamilton theorem, find the inverse of

$$A \quad \begin{matrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix}$$ 5

(b) Reduce the matrix

$$A \quad \begin{matrix} 0 & 1 & 2 & 2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{matrix}$$

to the normal form and hence determine its rank. 5

UNIT—II

3. *(a)* Define a binary operation. Let Q be the set of all positive rational numbers and a binary operation on Q defined by $a \quad b \quad \frac{ab}{3}$. Determine the identity element in Q and the inverse of a in Q. 1+2+2=5

(b) Prove that every proper subgroup of an infinite cyclic group is infinite. 5

4. *(a)* Prove that a necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup is that $a, b \quad H$ which implies $ab^{\ \ 1} \quad H$, where $b^{\ \ 1}$ is the inverse of b in G. 5

(b) Show that four fourth roots of unity $\{1, \quad 1, i, \quad i\}$ form a group with respect to multiplication. 5

UNIT—III

5. *(a)* State and prove Lagrange's theorem on the order of the group. 1+5=6

(b) Show that every group of prime order is cyclic. 4

6. *(a)* State and prove Fermat's theorem. 1+5=6

(b) If f is a homomorphism of a group G into a group G^1, prove that $f(a^{\ \ 1}) \quad [f(a)]^{\ \ 1}$ for all $a \quad G$. 4

UNIT—IV

7. *(a)* State and prove division algorithm. 1+5=6

(b) Expand $x^5 \quad 6x^3 \quad x^2 \quad 1$ in the powers of $(x \quad 1)$. 4

8. *(a)* If the equation $f(x) \quad 0$ has all its roots real, show that the equation $f(x)\ f\ \ (x) \quad [f\ \ (x)]^2 \quad 0$ has all its roots imaginary. 5

(b) Find the remainder, when $x^4 \quad 3x^3 \quad 4x^2 \quad 8x \quad 5$ is divided by $(x \quad 1)(x \quad 3)$. 5

UNIT—V

9. *(a)* Prove that the equation $x^4 \quad 15x^2 \quad 7x \quad 11 \quad 0$ has two real roots—one positive and the other negative and two other complex roots. 5

(b) If , , are the roots of the equation $x^3 \quad px^2 \quad qx \quad r \quad 0$, then find the equation whose roots are $\frac{1}{\ }$, $\frac{1}{\ }$, $\frac{1}{\ }$. 5

10. *(a)* If the equation $3x^4 \quad 4x^3 \quad 60x^2 \quad 96x \quad k \quad 0$ has four real and unequal roots, show that k must lie between 32 and 43. 5

(b) Using Cardan's method, solve the equation $x^3 \quad 12x^2 \quad 6x \quad 10 \quad 0$. 5

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MATH/II/EC/02 **Student's Copy**

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(CBCS)

(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Answer **all** questions

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The identity element of the group of all positive rational numbers under the composition $a \quad b \quad \frac{ab}{2}$ is

(a) 1 ( )

(b) 2 ( )

(c) 0 ( )

(d) 2 ( )

2. The number of generators of a cyclic group of order 8 is

(a) 2 ( )

(b) 7 ( )

(c) 4 ( )

(d) 8 ( )

3. If f is a homomorphism of G into G, then the set K of all those elements of G which are mapped by f onto the identity element of G is called

(a) kernel of the homomorphism f ( )

(b) homomorphism f ( )

(c) kernel of the isomorphism f ( )

(d) isomorphism f ( )

4. When 99^{20} is divided by 25, then the remainder is

(a) 20 ( ) (b) 15 ( )

(c) 5 ( ) (d) 1 ( )

5. If $f(x)$ and $g(x)$ be two polynomials of degrees m and n respectively, then $f(x).g(x)$ is a polynomial of degree

(a) $m.n$ ( ) (b) $m \quad n$ ( )

(c) m/n ( ) (d) n/m ( )

6. The value of the remainder, when $x^3 \quad 5x^2 \quad 1$ is divided by $x \quad 3$, is

(a) 18 ( ) (b) 19 ( )

(c) 27 ( ) (d) 19 ( )

7. The equation $x^4 \quad 3x^2 \quad 2x \quad 7 \quad 0$ has

(a) one positive, one negative and two imaginary roots ( )

(b) two positive and two negative roots ( )

(c) one positive and three negative roots ( )

(d) three positive and one negative root ( )

8. If $f(x)$ be a polynomial and $(x \quad a)$ is a factor of $f(x)$, then $f(a)$ equals to

(a) 1 ( ) (b) a ( )

(c) 0 ( ) (d) None of the above ( )

9. If , , be the roots of $x^3 \quad x \quad 1 \quad 0$, then the value of $^2 \quad ^2 \quad ^2$ is

(a) 0 ( ) (b) 2 ( )

(c) 1 ( ) (d) 2 ( )

10. If the sum of two roots of the equation $x^3 \quad 5x^2 \quad 16x \quad p \quad 0$ is zero, then the value of p is

(a) 16 ( ) (b) 80 ( )

(c) 0 ( ) (d) None of the above ( )

SECTION—B

(*Marks* : 15)

Answer **one** question from each Unit

Each question carries *3* marks

Unit—I

1. If a and b be any two elements of a group G and H be a subgroup of G, then prove that $Ha \quad Hb \quad ab^{\ 1} \quad H$.

2. Prove that the intersection of two subgroups of a group is a subgroup.

Unit—II

3. If $f : G \quad G$ is a group homomorphism, then prove that $f(a^{\quad 1}) \quad [f(a)]^{\quad 1}, \quad a \quad G$.

4. If G is a finite cyclic group of prime order, then prove that G has no proper subgroup.

Unit—III

5. Show that $x^4 \quad 3x^3 \quad 2x^2 \quad 2x \quad 4$ is divisible by $x \quad 2$.

6. If   be a root of the polynomial equation $f(x) \quad 0$, then prove that $(x \quad\quad)$ is a factor of $f(x)$.

Unit—IV

7. Prove that $x^4 \quad 16x^2 \quad 7x \quad 10 \quad 0$ has exactly two imaginary roots.

8. Write the equation of lowest degree with real coefficient having 2 and $(1 \quad 3i)$ as two of its roots.

Unit—V

9. If the equation $ax^3 \quad 3bx^2 \quad 3cx \quad d \quad 0$ has two equal roots, then $(bc \quad ad)^2 \quad 4(b^2 \quad ac)(c^2 \quad bd)$. (Write True or False)

Justification :

10. If   is any root of $x^n \quad 1 \quad 0$, then $\quad^m$ is also a root, where m is an integer. (Write True or False)

Justification :

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

Unit—I

1. *(a)* Let A be a set of the fourth root of unity, i.e., $A \quad \{1, \quad 1, i, \quad i\}$. Then show that the algebraic structure $(A, \quad)$ is a group, where   is an ordinary multiplication. 5

(b) Prove that every finite group of composite order possesses proper subgroups. 5

2. *(a)* Prove that every group of prime order is cyclic. 5

(b) Prove that every cyclic group is an Abelian group. 3

(c) Show that in a cyclic group G, if a is a generator of G, then $a^{\quad 1}$ is also a generator. 2

Unit—II

3. *(a)* State and prove Lagrange's theorem on the order of a group. 1+5=6

(b) State Euler's extension of Fermat's theorem and apply it to show that the remainder on dividing 11^7 by 18 is 11. 2+2=4

4. *(a)* Define kernel of a group homomorphism. If $f : G \quad G$ be a group homomorphism, prove that ker $f \quad \{e\}$ if and only if f is one-one. 1+4=5

(b) Show that the mapping $f : (R, \quad) \quad (R \;, \;)$ defined by $f(x) \quad e^x, \quad x \quad R$ is an isomorphism of R onto R . 5

Unit—III

5. *(a)* Find the remainder, by synthetic division, when the polynomial $3x^4 \quad 4x^3 \quad 2x^2 \quad 9x \quad 1$ is divided by $2x \quad 1$. 5

(b) If a polynomial $f(x)$ of degree $n \quad 2$ is divided by $(x \qquad)^2$, then prove that the remainder is $(x \qquad) f(\;) \quad f(\;)$. 5

6. *(a)* State and prove division algorithm on a polynomial. 1+5=6

(b) Prove that $x^2 \quad x \quad 1$ is a factor of $x^{10} \quad x^5 \quad 1$. 4

Unit—IV

7. *(a)* Solve the equation $x^4 \quad 7x^3 \quad 27x^2 \quad 47x \quad 26 \quad 0$, given that one of its roots is $2 \quad 3i$. 5

(b) Apply Descartes' rule of sign, examine the nature of the roots of the equation $x^4 \quad 2x^2 \quad 3x \quad 1 \quad 0$. 5

8. *(a)* Find the number and position of the real roots of $f(x) \quad 2x^5 \quad 4x^4 \quad 9x \quad 2 \quad 0$ by using Rolle's theorem. 5

(b) Find the range of value of k for which the equation $x^4 \quad 26x^2 \quad 48x \quad k \quad 0$ has four unequal roots. 5

Unit—V

9. *(a)* Solve $x^3 \quad 4x^2 \quad 3x \quad 18 \quad 0$, given that two of its roots are equal. 5

(b) Find all the values of $(\sqrt{3} \quad i)^{1/6}$ by De Moivre's theorem. 5

10. *(a)* Solve the equation $x^3 \quad 6x \quad 2 \quad 0$ by Cardan's method. 6

(b) If , , are roots of the equation $x^3 \quad qx \quad r \quad 0$, then form the equation whose roots are —, —, —. 4

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