MATH/II/02

2016

(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. (a) State and prove Cayley-Hamilton theorem for a square matrix. 1+4=5

(b) Obtain the fully reduced normal form of the matrix

$$\begin{matrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{matrix}$$

and hence find its rank. 5

2. (a) Using elementary operation, find the inverse of the matrix

$$\begin{matrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{matrix}$$

4

(b) Show that the equations

$$\begin{matrix} 5x & 3y & 7z & 4 \\ 3x & 26y & 2z & 9 \\ 7x & 2y & 10z & 5 \end{matrix}$$

have infinite number of solutions and solve the equations. 6

UNIT—II

3. (a) Show that the set I of all integers with binary operation defined by $a \quad b \quad a \quad b \quad 1$ forms a group. 5

(b) Prove that every finite group of composite order possesses proper subgroups. 5

4. (a) Prove that every cyclic group is an Abelian group. 2

(b) Prove that every proper subgroup of an infinite cyclic group is infinite. 3

(c) Prove that any two right cosets of a subgroup are either disjoint or identical. 5

UNIT—III

5. *(a)* State and prove Lagrange's theorem on the order of a group. 1+5=6

(b) Let $(G, \circ)$ and $(G ,\ \ )$ be two groups and $\ : G \quad G$ be a homomorphism. If K be a subgroup of G , then prove that $\ ^{1}(K\ )$ is a subgroup of G. 4

6. *(a)* Define kernel of a group of homomorphism. If $\ : G \quad G$ be a group homomorphism, then prove that $\ $ is one-to-one, if and only if ker $\quad \{e_G\}$. 1+3=4

(b) Show that the mapping $f : (R \ ,\ ) \quad (R,\ \ )$ defined by $f(x) \quad \log x$ forms a group isomorphism. 4

(c) Find the remainder of 3^{47} when divided by 23. 2

UNIT—IV

7. *(a)* State and prove division algorithm on a polynomial. 1+5=6

(b) If a polynomial $f(x)$ of degree $n \quad 2$ is divided by $(x \quad)^2$, then prove that the remainder is $(x \quad) f\ (\ ) \quad f(\ )$. 4

8. *(a)* Find the value of K for which the polynomial $4x^3 \quad 3x^2 \quad 2x \quad K$ is divisible by $(x \quad 2)$. 4

(b) Find the remainder when

$$x^5 \quad 5x^4 \quad x^3 \quad 5x^2 \quad 2x \quad 11$$

is divided by $x \quad 5$. 4

(c) If $x^3 \quad 3px \quad q$ has a factor of the form $(x \quad)^2$, then show that $q^2 \quad 4p^3 \quad 0$. 2

UNIT—V

9. *(a)* Using Descartes' rule of sign, examine the nature of the roots of the equation $x^6 \quad x^4 \quad x^2 \quad x \quad 3 \quad 0$. 4

(b) Solve the equation $x^3 \quad 3x^2 \quad 9x \quad 14 \quad 0$ by Cardan's method. 6

10. *(a)* Find all the values of $(1 \quad i)^{1/7}$ by De Moivre's theorem. 5

(b) If $\ ,\ ,\ $ are the roots of the equation $x^3 \quad px^2 \quad qx \quad r \quad 0$, then form the equation whose roots are

$$\frac{}{},\ \frac{}{},\ \frac{}{}$$ 5

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Subject Code : MATH/II/02

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 2nd Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....

Paper ....

To be filled in by the Candidate

DEGREE 2nd Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

Signature of Scrutiniser(s) *Signature of Examiner(s)* *Signature of Invigilator(s)*

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MATH/II/02

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(2nd Semester)

MATHEMATICS

SECOND PAPER

(Algebra)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If $S \quad M \quad iN$ be a skew-Hermitian matrix, then the diagonal elements of S are all

(a) real numbers ☐

(b) 1 ☐

(c) imaginary numbers or zero ☐

(d) None of the above ☐

2. The rank of the matrix

$$A \quad \begin{matrix} 1 & 0 & 3 \\ 4 & 1 & 5 \\ 2 & 0 & 6 \end{matrix}$$

is

(a) 2 ☐

(b) 1 ☐

(c) 0 ☐

(d) 3 ☐

3. The identity element of the group of all positive rational numbers under the composition $a \quad b \quad \frac{ab}{2}$ is

(a) –2 ☐

(b) 1 ☐

(c) 0 ☐

(d) 2 ☐

4. The number of generators of a cyclic group of order 8 is

(a) 2 ☐

(b) 4 ☐

(c) 7 ☐

(d) 8 ☐

5. When 45^{16} is divided by 32, then the remainder is

(a) 1 ☐

(b) 32 ☐

(c) 44 ☐

(d) 16 ☐

6. A homomorphism $f : G \quad G$ is said to be an isomorphism, if f is

(a) one-to-one mapping ☐

(b) into mapping ☐

(c) one-to-one and into mapping ☐

(d) one-to-one and onto mapping ☐

7. If $f(x)$ and $g(x)$ be two polynomials of degree n and m respectively, then degree of $f(x) \quad g(x)$ is

(a) m / n ☐

(b) $m \quad n$ ☐

(c) $m \quad n$ ☐

(d) $m \quad n$ ☐

8. If $f(x)$ be a polynomial and $(x \quad a)$ is a factor of $f(x)$, then $f(a)$ equals to

(a) 1 ☐

(b) 0 ☐

(c) a ☐

(d) None of the above ☐

9. The equation $x^7 \quad x^5 \quad x^3 \quad 0$ has

(a) five real roots and two complex roots ☐

(b) two real roots and five complex roots ☐

(c) one positive root, two negative roots and four complex roots ☐

(d) None of the above ☐

10. If , , , be the roots of the biquadratic equation $a_0x^4 \quad a_1x^3 \quad a_2x^2 \quad a_3x \quad a_4 \quad 0$, then equals to

(a) $\frac{a_1}{a_0}$ ☐

(b) $\frac{a_2}{a_0}$ ☐

(c) $\frac{a_3}{a_0}$ ☐

(d) $\frac{a_3}{a_0}$ ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

State *True* or *False* of the following with a brief justification :

1. If A be an orthogonal matrix, then $A^{\ 1}$ is orthogonal.

True ☐ *False* ☐

Justification :

2. If every element of a group is its own inverse, then *G* is Abelian.

True ☐ *False* ☐

Justification :

3. Every isomorphic image of a cyclic group is cyclic.

True ☐ *False* ☐

Justification :

4. If a polynomial $f(x)$ is divided by a binomial $(x \quad a)$, then the remainder is $f(\ \ a)$.

True ☐ *False* ☐

Justification :

5. If the equation $ax^3 \quad 3bx^2 \quad 3cx \quad d \quad 0$ has two equal roots, then $(bc \quad ad)^2 \quad 4(b^2 \quad ac)(c^2 \quad bd)$.

True ☐ *False* ☐

Justification :

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