

2 0 1 5

(1st Semester)

MATHEMATICS

FIRST PAPER

(Calculus)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(Marks : 50)

The figures in the margin indicate full marks for the questions

Answer any **one** question from each Unit

UNIT—I

1. (a) Draw the graph of the function $y = x [x]$ in the interval $[-3, 3]$, where $[x]$ denote the greatest integer function. 4

(b) Evaluate : 3

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x \sin x}$$

(c) Examine the continuity of the function

$$f(x) = \begin{cases} \frac{x|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases} \quad 3$$

2. (a) The relation between volume (V) and pressure (P) of a gas is given by $V = \frac{200}{P}$.

Find the average rate of change of volume with respect to pressure when P increases from 30 to 35. Also find the instantaneous rate of change of volume at $P = 30$. 5

(b) If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 y_n - 2(n-1)xy_{n-1} + (n^2-1)y_{n-2} = 0$ 5

UNIT—II

3. (a) State and prove Maclaurin's theorem. 6

(b) Show that the length of the portion of the tangent to the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ intercepted between the coordinate axes is constant. 4

4. (a) Let f be defined and continuous on $[a-h, a+h]$ and derivable on $]a-h, a+h[$. Prove that there is a real number θ between 0 and 1 for which $f(a+h) - f(a-h) = h\{f'(a+\theta h) - f'(a-\theta h)\}$ 4

(3)

- (b) Expand e^x in the power of $(x - 5)$. 4
- (c) Is Rolle's theorem valid in $[-1, 1]$ for the function $f(x) = x^2$? Justify your answer. 2

UNIT—III

5. (a) Evaluate $\int_1^2 \frac{1}{x^2} dx$ using definite integral as a limit of sum. 5
- (b) Obtain reduction formula for the integral $\int \cos^n x dx$ 5

6. (a) Evaluate : 3
- $$\int_0^{1/2} \frac{dx}{1 + \sqrt{\cot x}}$$

- (b) Obtain reduction formula for $\int e^{ax} x^n dx$ 3

- (c) Integrate any one of the following : 4

(i) $\int \frac{dx}{x(x^7 - 1)}$

(ii) $\int e^x \frac{x^2 - 1}{(x + 1)^2} dx$

(4)

UNIT—IV

7. (a) Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function given by

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Test the continuity of f at $(0, 0)$. 5

- (b) If $u = \log \sqrt{x^2 + y^2 + z^2}$, prove that

$$(x^2 + y^2 + z^2) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 1$$
 5

8. (a) Evaluate $\int_C xy^4 ds$ where C is the right half of the circle, $x^2 + y^2 = 16$ rotated in the counter clockwise direction. 5

- (b) Change the order of integration

$$I = \int_0^1 \int_{x^2}^2 xy dx dy$$

and hence evaluate it. 5

UNIT—V

9. (a) Show that the sequence $\{f_n\}$ defined by

$$f_n = 1 - \frac{1}{2} + \frac{1}{3} - \dots + \frac{1}{n}$$

cannot converge. 5

(5)

(b) Prove that a monotonically increasing sequence bounded above is convergent. 5

10. (a) Prove that the positive term series $\frac{1}{n^p}$ is convergent if and only if $p > 1$. 5

(b) Show that

$$1 + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 4 \cdot 5} + \frac{1}{2 \cdot 4 \cdot 6 \cdot 7} + \dots$$

converges by Raabe's test. 5

Subject Code : **I**/MAT (i)

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Booklet No. **A**

Date Stamp

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To be filled in by the Candidate

DEGREE 1st Semester
(Arts / Science / Commerce /
.....) Exam., **2015**
Subject
Paper

.....

To be filled in by the Candidate

DEGREE 1st Semester
(Arts / Science / Commerce /
.....) Exam., **2015**

Roll No.

Regn. No.

Subject

Paper

Descriptive Type

Booklet No. B

INSTRUCTIONS TO CANDIDATES

- 1. The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
- 2. This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
- 3. While answering the questions of this booklet, any cutting, erasing, overwriting or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

*Signature of
Scrutiniser(s)*

*Signature of
Examiner(s)*

*Signature of
Invigilator(s)*

2 0 1 5

(1st Semester)

MATHEMATICS

FIRST PAPER

(**Calculus**)

(PART : A—OBJECTIVE)

(Marks : 25)

SECTION—I

(Marks : 10)

Each question carries 1 mark

Answer **all** questions

Put a Tick mark against the correct answer in the box provided :

1. The function $f(x) = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$ is

(a) not continuous at $x = 0$

(b) continuous and derivable at $x = 0$

(c) continuous but not derivable at $x = 0$

(d) None of the above

(2)

2. The n th derivative of $5x^n$ is

(a) $5n(n-1)x^{n-1}$

(b) $(5n)!$

(c) $5n!$

(d) $5x(n-1)!$

3. If $f'(x) = 0$ at $x = c$, i.e., $f'(c) = 0$, then

(a) $f(x)$ is increasing at $x = c$

(b) $f(x)$ is decreasing at $x = c$

(c) $f(x)$ is neither increasing nor decreasing at $x = c$

(d) $x = c$ is the critical point

4. The slope of the normal to the curve $y = f(x)$ at the point (a, b) is

(a) $\frac{dy}{dx} (a, b)$

(b) $\frac{dx}{dy} (a, b)$

(c) $\frac{1}{\frac{dx}{dy} (a, b)}$

(d) $\frac{1}{\frac{dy}{dx} (a, b)}$

(3)

5. The value of the integral $\int_1^1 [x] dx$ is

(a) 1

(b) 2

(c) 0

(d) -1

6. The value of $\int_1^1 x^{777} dx$ is equal to

(a) 0

(b) 1

(c) $\frac{1}{778}$

(d) $\frac{1}{778}$

7. The value of the double integral $\int_0^2 \int_1^3 xy \, dy \, dx$ is

(a) 0

(b) 4

(c) 8

(d) 16

(4)

8. The function $f(x, y) = \sin^{-1} \frac{x^2 - y^2}{x + y}$ is

(a) homogeneous of degree 1

(b) homogeneous of degree 2

(c) homogeneous of degree 3

(d) not homogeneous

9. The sequence $\{r^n\}$

(a) converges to 1 if $r > 1$

(b) converges to 1 if $r < 1$

(c) converges to 1 if $r = 1$

(d) converges to 0 if $r < 1$

10. The series $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \dots$ is

(a) convergent

(b) not convergent

(c) oscillatory

(d) None of the above

(5)

SECTION—II

(Marks : 15)

Each question carries 3 marks

Answer **all** questions

1. Show that $f(x) = x^3$ is derivable on $[0, 1]$.

(6)

2. State Rolle's theorem and explain its geometrical interpretation.

(7)

3. Evaluate :

$$e^x \frac{x-1}{(x-1)^3} dx$$

(8)

4. Does $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$ exist? Justify your answer.

(9)

5. Prove that every convergent sequence is bounded.
