

2017

( 4th Semester )

MATHEMATICS

Paper : MATH-241

( Vector Calculus and Solid Geometry )

Full Marks : 75

Time : 3 hours

( PART : B—DESCRIPTIVE )

( Marks : 50 )

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. (a) Prove that for a triangle ABC,
- $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
- where BC = a, CA = b and AB = c. 4
- (b) Find the set of vectors reciprocal to the set  $2\hat{i} + 3\hat{j} + \hat{k}$ ,  $\hat{i} + \hat{j} + 2\hat{k}$  and  $\hat{i} + 2\hat{j} + 2\hat{k}$ . 4

- (c) Find the unit tangent vector to the space curve  $\vec{r} = t\hat{i} + t^2\hat{j} + \frac{2}{3}t^3\hat{k}$  at  $t = 1$ . 2
2. (a) Prove that the necessary and sufficient condition for the vector function  $\vec{f}(t)$  to have constant direction is  $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$ . 5
- (b) If three concurrent edges of a parallelepiped are given by  $\vec{a} = 3\hat{i} + 7\hat{j} + 5\hat{k}$ ,  $\vec{b} = 5\hat{i} + 7\hat{j} + 3\hat{k}$  and  $\vec{c} = 7\hat{i} + 5\hat{j} + 3\hat{k}$  then find its volume. 5

UNIT—II

3. (a) Show that  $\nabla f(r) = \frac{\vec{r}}{r}$ , where  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  and  $r = |\vec{r}|$ . 5
- (b) Let  $(x, y, z) = 3x^2y + y^2z^2$ . Find the directional derivative of at (1, 2, 1) in the direction  $\hat{i} + \hat{j} + \hat{k}$  and the greatest directional derivative at that point. 4+1
4. (a) Find the work done in moving a particle from (0, 0) to (1, 2) along the curve  $y = 2x^2$  in  $xy$ -plane, where the force field is  $\vec{F} = 3xy\hat{i} + y^2\hat{j}$ . 3

( 3 )

- (b) Show that  $\text{curl grad } \vec{0}$ , where  $\phi$  is a scalar function of  $x, y$  and  $z$ . 3
- (c) Verify Stokes' theorem for the function  $\vec{F} = x^2\hat{i} + xy\hat{j}$  integrated along the rectangle in the  $xy$ -plane whose sides are along the lines  $x = 0, y = 0; x = a$  and  $y = b$ . 4

UNIT—III

5. (a) Find the transformed equation of the curve  $(x - 2y - 4)(2x - y - 5) = 25$ , when the two perpendicular lines  $x - 2y = 0$  and  $2x - y = 0$  are taken as coordinate axes. 5
- (b) Find the values of  $k$  for which the equation  $3x^2 + kxy + 3y^2 - 29x - 3y - 18 = 0$  represents a pair of straight lines. 5
6. (a) Reduce the equation  $7x^2 - 2xy + 7y^2 - 16x - 16y - 8 = 0$  to the standard form and hence show that it is the equation of an ellipse. 6

( 4 )

- (b) Find the pair of tangents from the point  $(1, 1)$  to the conic  $14x^2 - 4xy + 11y^2 - 44x + 58y - 71 = 0$ . 4

UNIT—IV

7. (a) Show that the lines  $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-1}{2}$  and  $\frac{x-3}{2} = \frac{y}{2} = \frac{z-2}{1}$  are coplanar. 4
- (b) Find the equations of the three planes through the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  parallel to the axes. 3
- (c) Find the equation of the plane passing through the points  $(4, 0, -2), (1, 1, -1)$  and origin. 3
8. (a) Find the equations of the planes bisecting the angles between the planes  $x - 2y + 2z - 3 = 0$  and  $3x + 4y + 12z - 1 = 0$ . 4

( 5 )

- (b) Prove that the equation of the line through the point  $(x_1, y_1, z_1)$  perpendicular to the plane  $ax + by + cz + d = 0$  is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

and find the perpendicular distance of the point  $(x_1, y_1, z_1)$  from the plane  $ax + by + cz + d = 0$ . 6

( 6 )

- (b) Prove that the locus of the point from which three mutually perpendicular lines can be drawn to intersect the conic  $z = 0, ax^2 + by^2 = 1$  is given by  $ax^2 + by^2 - (a + b)z^2 = 1$ . 5

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UNIT—V

9. (a) Find the radius and centre of the circle where the plane  $x + 2y + 2z = 3$  intersects the sphere

$$x^2 + y^2 + z^2 + 8x + 4y + 8z = 45 \quad 5$$

- (b) Find the equations of the tangent planes to the sphere

$$x^2 + y^2 + z^2 + 2x + 4y + 6z + 2 = 0$$

parallel to the plane  $x + y + z = 0$ . 5

10. (a) Find the equation of the cylinder generated by the lines parallel to the line  $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$  and intersecting the guiding curve  $z = 3, x^2 + y^2 = 4$ . 5

Subject Code : MATH/IV/04

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Booklet No. **A**

Date Stamp .....

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**To be filled in by the Candidate**

DEGREE 4th Semester  
(Arts / Science / Commerce /  
..... ) Exam., **2017**  
Subject .....  
Paper .....

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DEGREE 4th Semester  
(Arts / Science / Commerce /  
..... ) Exam., **2017**

Roll No. ....

Regn. No. ....

Subject .....

Paper .....

Descriptive Type

Booklet No. B .....

**INSTRUCTIONS TO CANDIDATES**

1. The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.
2. This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.
3. While answering the questions of this booklet, any cutting, erasing, overwriting or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.

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( 4th Semester )

**MATHEMATICS**

Paper : MATH-241

**( Vector Calculus and Solid Geometry )**

( PART : A—OBJECTIVE )

( Marks : 25 )

Answer **all** questions

SECTION—I

( Marks : 10 )

*Each question carries 1 mark*

Put a Tick  mark against the correct answer in the box provided :

1. If  $\vec{a}$  and  $\vec{b}$  are non-zero vectors and  $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}|$ , then  $\vec{a}$  and  $\vec{b}$  are

(a) perpendicular to each other

(b) parallel to each other

(c) neither parallel nor perpendicular

(d) None of the above

( 2 )

2. If  $\vec{f}(t)$  be a vector valued function with constant direction, then

(a)  $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$

(b)  $\vec{f} \cdot \frac{d\vec{f}}{dt} = \vec{0}$

(c)  $\left| \vec{f} \cdot \frac{d\vec{f}}{dt} \right| = \left| \vec{f} \right| \left| \frac{d\vec{f}}{dt} \right|$

(d) None of the above

3. If the vector  $\vec{V} = y^2z\hat{i} + axyz\hat{j} + xy^2\hat{k}$  be a conservative vector, then  $a$  is equal to

(a) 0

(b) 2

(c) 1

(d) None of the above

4. Let  $C$  be the unit circle in  $xy$ -plane with centre at origin. Then  $\oint_C y dx - x dy$  is equal to

(a) 2

(b)

(c) 0

(d) None of the above

( 3 )

5. By changing the origin to (2, 3) without changing the direction of axes the equation  $5x - 3y - 3 = 0$  changes to

(a)  $5x - 3y - 1 = 0$

(b)  $5x - 3y - 6 = 0$

(c)  $5x - 3y - 16 = 0$

(d) None of the above

6. The equation  $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$  represents

(a) an ellipse

(b) a parabola

(c) a hyperbola

(d) None of the above

7. The angle of inclination of the line  $x - y = 0, z = 0$  with  $z$ -axis is

(a)  $45^\circ$

(b)  $60^\circ$

(c)  $90^\circ$

(d) None of the above

8. The shortest distance between the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and the  $x$ -axis is

- (a)  $\frac{2}{5}$
- (b)  $\frac{1}{5}$
- (c) 5
- (d) None of the above
9. The general equation of the sphere which touches the  $xy$ -plane at origin is
- (a)  $x^2 + y^2 + z^2 - 2ux = 0$
- (b)  $x^2 + y^2 + z^2 - 2vy = 0$
- (c)  $x^2 + y^2 + z^2 - 2wz = 0$
- (d) None of the above
10. The equation of the cone with its vertex at origin and direction cosines of its generators satisfying the relation  $4l^2 + 5m^2 + 7n^2 = 0$  is
- (a)  $4x^2 + 5y^2 + 7z^2 = 0$
- (b)  $4x^2 - 5y^2 - 7z^2 = 0$
- (c)  $4x^2 - 5y^2 + 7z^2 = 0$
- (d) None of the above



( 5 )

SECTION—II

( Marks : 15 )

*Each question carries 3 marks*

State *True* or *False* by putting a Tick  mark in the box provided and give a brief justification :

1. The set of vectors  $\hat{i}, \hat{j}, \hat{k}$  and  $\hat{k}, \hat{i}$  has reciprocal set of vectors.

*True*      *False*  

*Justification :*

( 6 )

2. The vector

$$\vec{V} = (4x - 3y + 4z)\hat{i} + (3x + 3y + 5z)\hat{j} + (4x + 5y + 3z)\hat{k}$$

is irrotational.

True  False

*Justification :*

( 7 )

3. The pair of straight lines  $x^2 - 5xy + 6y^2 = 0$  are perpendicular to the pair of straight lines  $6x^2 - 5xy - y^2 = 0$ .

True  False

*Justification :*

( 8 )

4. The line

$$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z}{1}$$

wholly lies on the plane  $x - 2y - z = 3$ .

*True*  *False*

*Justification :*

( 9 )

5. The line joining the centre of the sphere  $x^2 + y^2 + z^2 = a^2$  to any point  $P$  is perpendicular to the polar plane of  $P$  with respect to the sphere  $x^2 + y^2 + z^2 = a^2$ .

*True*      *False*  

*Justification :*

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