2 0 1 7

(1st Semester)

MATHEMATICS

FIRST PAPER

(Calculus—I)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Draw the graph of the function defined by

$$f(x) \quad \begin{matrix} x & , & 0 & x & 1 \\ 2 & x & , & 1 & x & 2 \\ x & 2 & , & 2 & x & 3 \end{matrix}$$

Discuss whether $f(x)$ exists at $x \quad 2$. 5

(b) Use - definition of continuity to prove that $y \quad \sin x$ is continuous at every value of x. 5

2. *(a)* If $y \quad e^{\tan^{1} x}$, then prove that

$$(1 \quad x^2)\, y_{n \quad 1} \quad (2nx \quad 1) y_n \quad n(n \quad 1) y_{n \quad 1} \quad 0$$ 5

(b) Evaluate : 3

$$\lim_{x \quad 0} \frac{e^x \quad e^{\sin x}}{x \quad \sin x}$$

(c) Prove that $f(x) \quad |x|$ is not differentiable at $x \quad 0$. 2

UNIT—II

3. *(a)* State and prove mean-value theorem and give its geometrical interpretation. 5

(b) Find the tangent to the curve

$$xy^2 \quad 4(4 \quad x)$$

at the point where it is cut by the line $y \quad x$. 5

4. *(a)* Expand $\log(1 \quad x)$ in an infinite series in powers of x. 5

(b) Find the value of in the Lagrange's form of remainder R_n for the expansion $\frac{1}{1 \quad x}$ in powers of x. 5

UNIT—III

5. *(a)* Prove that

$$\int_0^{\pi/2} \cos^n x\,dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$$

if n is even. 5

(b) From the definition of integration as the limit of sum, evaluate

$$\int_2^5 e^x\,dx$$

5

6. *(a)* Evaluate any *two* of the following : 2½×2=5

(i) $\displaystyle\int \frac{\sin x}{\sin x + \cos x}\,dx$

(ii) $\displaystyle\int \frac{(2x+5)}{\sqrt{x^2+3x+1}}\,dx$

(iii) $\displaystyle\int \frac{x-1}{(x-2)(x-3)}\,dx$

(b) Prove that

$$\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\cdots+\frac{1}{n}\right]=\frac{\pi}{4}+\frac{1}{2}\log 2$$

5

UNIT—IV

7. *(a)* Let $f:\mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y)=\begin{cases}\dfrac{x^2y}{x^2+y^2} & ;\ \text{if } (x,y)\neq(0,0)\\ 0 & ;\ \text{if } (x,y)=(0,0)\end{cases}$$

Test the continuity of f at $(0, 0)$. 5

(b) If

$$u=\tan^{-1}\frac{x^3+y^3}{x-y}$$

then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$$

5

8. *(a)* Show that

$$\int_0^1 dx\int_0^1\frac{x-y}{(x+y)^3}\,dy \neq \int_0^1 dy\int_0^1\frac{x-y}{(x+y)^3}\,dx$$

5

(b) Find the area of the portion of the circle $x^2+y^2=1$, which lies inside the parabola $y^2=1-x$. 5

UNIT—V

9. *(a)* Prove that a necessary and sufficient condition for the convergence of a sequence $\{S_n\}$ is that, for each 0 there exists a positive integer m such that

$$\left| S_{n \quad p} \quad S_n \right|$$

for every $n \quad m$ and $p \quad 1$. 5

(b) Show that the sequence $\{S_n\}$, where

$$S_n \quad \frac{1}{1!} \quad \frac{1}{2!} \quad \cdots \quad \frac{1}{n!}$$

is convergent. 3

(c) Prove that the sequence $\{S_n\}_{n \quad \mathbb{N}}$, where

$$S_n \quad \frac{3n \quad 1}{n \quad 2}$$

is bounded. 2

10. *(a)* If U_n is a positive term series such that

$$\lim_{n} \frac{U_{n \quad 1}}{U_n} \quad l$$

then show that—

(i) the series converges if $l \quad 1$;

(ii) the series diverges if $l \quad 1$;

(iii) the test fails if $l \quad 1$. 5

(b) Prove that the positive term geometric series $1 \quad r \quad r^2 \quad \cdots$ converges for $r \quad 1$ and diverges to for $r \quad 1$. 3

(c) Test the convergence of the series

$$\sum_{n \quad 1} \{(n^3 \quad 1)^{1/3} \quad n\}$$

2

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Subject Code : MATH/I/01

Booklet No. A

Date Stamp ....

....

To be filled in by the Candidate

DEGREE 1st Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

1. **The Booklet No. of this script should be quoted in the answer script meant for descriptive type questions and vice versa.**
2. **This paper should be ANSWERED FIRST and submitted within 1 (one) Hour of the commencement of the Examination.**
3. **While answering the questions of this booklet, any cutting, erasing, over-writing or furnishing more than one answer is prohibited. Any rough work, if required, should be done only on the main Answer Book. Instructions given in each question should be followed for answering that question only.**

To be filled in by the Candidate

DEGREE 1st Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

Signature of Scrutiniser(s)

Signature of Examiner(s)

Signature of Invigilator(s)

MATH/I/01

2 0 1 7

(1st Semester)

MATHEMATICS

FIRST PAPER

(Calculus—I)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—I

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The limit $\lim_{x \to 0} \frac{2^{3x} - 1}{x}$ is equal to

(a) $2\log_e 3$ ☐

(b) $3\log_e 2$ ☐

(c) $\frac{1}{3}\log_e 2$ ☐

(d) $\frac{1}{2}\log_e 3$ ☐

2. If $y \quad Ae^{2x} \quad Be^{\;2x}$, then $\dfrac{d^2y}{dx^2}$ is

(a) y ☐

(b) $x \quad y$ ☐

(c) $4y$ ☐

(d) None of the above ☐

3. The function $f(x) \quad \log(x \quad 1)$ can be expanded in power of x by using

(a) Rolle's theorem ☐

(b) Leibnitz's theorem ☐

(c) Taylor's theorem ☐

(d) Maclaurin's theorem ☐

4. Using mean-value theorem, the point to the curve $g \quad x^2$, where the tangent is parallel to the line joining the points (1, 1) and (2, 4) is

(a) $1, \dfrac{9}{4}$ ☐

(b) $\dfrac{3}{4}, \dfrac{9}{4}$ ☐

(c) $\dfrac{3}{2}, \dfrac{9}{4}$ ☐

(d) None of the above ☐

5. If f is an even function and $\int_0^1 f(x)\,dx = 4$, then the value of $\int_{-1}^{1} f(x)\,dx$ is

(a) 4 ☐

(b) -4 ☐

(c) 8 ☐

(d) -8 ☐

6. If $f(x) = [x + 1]$ (the greatest integer function), then the value of $\int_{-1}^{1} f(x)\,dx$ is equal to

(a) 0 ☐

(b) 1 ☐

(c) 2 ☐

(d) -1 ☐

7. The limit $\lim_{(x,\, y) \to (0,\, 0)} \dfrac{2xy}{x^2 + y^2}$ is equal to

(a) 2 ☐

(b) 0 ☐

(c) 1 ☐

(d) Does not exist ☐

8. The value of the integral $_C\ xy\,dx$ along the arc of a parabola $x \quad y^2$ from (1, 1) to (1, 1) is

(a) $\frac{1}{5}$ ☐

(b) 4 ☐

(c) $\frac{3}{5}$ ☐

(d) $\frac{4}{5}$ ☐

9. The sequence $\{n\,(\quad 1)^n\},\ n \quad \mathbb{N}$

(a) oscillates finitely ☐

(b) oscillates infinitely ☐

(c) always has a limit 0 ☐

(d) None of the above ☐

10. The series $\frac{n^2 \quad 1}{n^2 \quad 1} x^n, \ x \quad 0$

(a) converges for any $x \quad 1$ ☐

(b) converges for any $x \quad 0$ ☐

(c) diverges for any $x \quad 0$ ☐

(d) diverges for any $x \quad 1$ ☐

SECTION—II

(*Marks* : 15)

Each question carries 3 marks

1. Test the continuity of $f(x)$ at $x \quad \frac{1}{2}$, where

$$f(x) \quad \begin{array}{ll} x & , \text{ if } \; x \; \text{ is rational} \\ 1 \quad x & , \text{ if } \; x \; \text{ is irrational} \end{array}$$

2. Find the derivative of $\log_5 x$ with respect to $\sin^{\ 1} x^2$.

3. If

$$I_n = \int_0^{\pi/4} \tan^n x \, dx$$

then prove that

$$I_n = \frac{1}{n-1} - I_{n-2}$$

4. If $f(x, y)$ be a homogeneous function of x and y of degree n, then prove that

$$x \frac{f}{x} \quad y \frac{f}{y} \quad nf(x, y)$$

5. Prove that every convergent sequence is always bounded but the converse is not true. Justify with suitable example.

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