MATH/IV/04 **Student's Copy**

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(Pre-CBCS)

(4th Semester)

MATHEMATICS

Paper : MATH–241

(Vector Calculus and Solid Geometry)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Tick (✓) the correct answer in the brackets provided :

1. If $\vec{a}$ and $\vec{b}$ are two mutually perpendicular proper vectors, then $\vec{a} \quad (\vec{b} \quad \vec{a})$ is parallel to

(a) $\vec{a}$ ( )

(b) $\vec{b}$ ( )

(c) $\vec{a} \quad \vec{b}$ ( )

(d) None of the above ( )

2. If $|\vec{a}| \quad 4$, $|\vec{b}| \quad 5$ and $\vec{a} \quad \vec{b} \quad 0$, then $\vec{a} \quad \vec{b}$ is

(a) $20\hat{n}$ ( )

(b) $9\hat{n}$ ( )

(c) $\hat{n}$ ( )

(d) 0 ( )

3. The vector $\vec{V}\quad (\;4x\quad 6y\quad 3z)\hat{i}\quad (\;2x\quad y\quad 5z)\hat{j}\quad (5x\quad 6y\quad az)\hat{k}$ is solenoidal, then the value of a is

(a) 5 ( )

(b) 8 ( )

(c) 3 ( )

(d) None of the above ( )

4. Suppose V be the volume bounded by a closed surface S, $\vec{r}\quad x\hat{i}\quad y\hat{j}\quad z\hat{k}$ and $\hat{n}$ is the unit vector normal (outward) to the surface S, then

$$\underset{S}{\;}\ \vec{r}\quad \hat{n}dS$$

is

(a) 0 ( )

(b) $4V$ ( )

(c) $2V$ ( )

(d) $3V$ ( )

5. The equation of pair of straight lines through the origin perpendicular to the pair $ax^2\quad 2hxy\quad by^2\quad 0$ is

(a) $ax^2\quad 2hxy\quad by^2\quad 0$ ( )

(b) $bx^2\quad 2hxy\quad ay^2\quad 0$ ( )

(c) $ax^2\quad 2hxy\quad by^2\quad 0$ ( )

(d) $bx^2\quad 2hxy\quad ay^2\quad 0$ ( )

6. If the equation $ax^2\quad 2hxy\quad by^2\quad 2gx\quad 2fy\quad c\quad 0$ represents a circle, if

(a) $ab\quad h^2\quad 0$ ( )

(b) $ab\quad h^2\quad 0$ ( )

(c) $a\quad b$ and $h\quad 0$ ( )

(d) $a\quad b\quad 0$ ( )

7. The intercepts made on the axes by the plane $3x\quad 4y\quad 6z\quad 12\quad 0$ are

(a) 4, 3 and 2 ( )

(b) 4, 3 and 5 ( )

(c) 5, 7 and 9 ( )

(d) None of the above ( )

8. The shortest distance between the line $\frac{x\quad 1}{4}\quad \frac{y\quad 2}{3}\quad \frac{z\quad 3}{1}$ and z-axis is

(a) $\frac{12}{5}$ ( )

(b) $\frac{11}{\sqrt{5}}$ ( )

(c) $\frac{11}{5}$ ( )

(d) $\frac{12}{7}$ ( )

9. The equation of sphere which passes through the origin and makes equal intercepts of unit length of the axes is

(a) $x^2\quad y^2\quad z^2\quad 1$ ( )

(b) $(x\quad 1)^2\quad (y\quad 1)^2\quad (z\quad 1)^2\quad 0$ ( )

(c) $x^2\quad y^2\quad z^2\quad x\quad y\quad z\quad 1$ ( )

(d) $x^2\quad y^2\quad z^2\quad x\quad y\quad z\quad 0$ ( )

10. The condition that the plane $lx\quad my\quad nz\quad 0$ touches the cone $ax^2\quad by^2\quad cz^2\quad 0$ is

(a) $bcl^2\quad cam^2\quad abn^2\quad 0$ ( )

(b) $al^2\quad bm^2\quad cn^2\quad 0$ ( )

(c) $a^2l\quad b^2m\quad c^2n\quad 0$ ( )

(d) None of the above ( )

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

State *True* or *False* by putting a Tick (✓) mark in the brackets provided and give a brief justification :

1. If $\vec{a}\quad \hat{i}\quad 2\hat{j}\quad 3\hat{k}$, $\vec{b}\quad \hat{i}\quad 3\hat{j}\quad 5\hat{k}$ and $\vec{c}\quad \hat{i}\quad \hat{j}\quad 6\hat{k}$, then the value of $\vec{a}\quad (\vec{b}\quad \vec{c})$ is 5.

True ( ) *False* ( )

Justification :

2. The value of $\frac{1}{r}$, where $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ is $\frac{\vec{r}}{r^3}$.

True ( ) *False* ( )

Justification :

3. The equation of the diameter of the conic $4x^2 \quad 6xy \quad 5y^2 \quad 1$ conjugate to the diameter $y \quad 2x \quad 0$ is $10y \quad 7x \quad 0$.

True ( ) *False* ( )

Justification :

4. The equation of the plane through the line $x \quad y \quad z \quad 3 \quad 0, \ 2x \quad y \quad 3z \quad 1 \quad 0$ and parallel to the line $\frac{x}{1} \quad \frac{y}{2} \quad \frac{z}{3}$ is $x \quad 5y \quad 3z \quad 7 \quad 0$.

True ( ) *False* ( )

Justification :

5. The equation of the orthogonal projection of the curve $2x \quad y \quad z \quad 3$, $x^2 \quad 2y^2 \quad 3z^2 \quad 1$ on the z-plane is

$$z \quad 0, 13x^2 \quad 5y^2 \quad 36x \quad 18y \quad 12xy \quad 26 \quad 0$$

True ( ) *False* ( )

Justification :

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

Unit—I

1. *(a)* Find a unit vector perpendicular to the plane of $\vec{A} \quad 2\hat{i} \quad 6\hat{j} \quad 3\hat{k}$ and $\vec{B} \quad 4\hat{i} \quad 3\hat{j} \quad \hat{k}$. 3

(b) If ABC be a triangle, then prove that

$$\cos A \quad \frac{b^2 \quad c^2 \quad a^2}{2bc}$$ 3

(c) Find the set of vectors reciprocal to the set $\hat{i} \quad 2\hat{j} \quad 3\hat{k}$, $5\hat{i} \quad \hat{j} \quad \hat{k}$ and $\hat{i} \quad \hat{j} \quad \hat{k}$. 4

2. (a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then prove that $[\vec{a} \quad \vec{b} \;\; \vec{b} \quad \vec{c} \;\; \vec{c} \quad \vec{a}] \quad [\vec{a}\ \vec{b}\ \vec{c}]$. 5

(b) If three concurrent edges of a parallelepiped is given by $\vec{a} \quad 2\hat{i} \quad 3\hat{j} \quad 4\hat{k}$, $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$ and $\vec{c} \quad 3\hat{i} \quad \hat{j} \quad 2\hat{k}$, then find its volume. 5

Unit—II

3. (a) If $(x, y, z) \quad x^2yz \quad 4xyz^2$, then find the directional derivative of in the direction of $\vec{A} \quad 2\hat{i} \quad \hat{j} \quad 2\hat{k}$ at (1, 3, 1). 4

(b) Prove that curl$(\vec{a} \quad \vec{b}) \quad (\vec{b} \quad)\vec{a} \quad \vec{b}(\quad \vec{a}) \quad (\vec{a} \quad)\vec{b} \quad \vec{a}(\quad \vec{b})$. 6

4. (a) If $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$, then show that $r^n \quad nr^{n \quad 2}\vec{r}$. 4

(b) Show that $_S \vec{F} \quad \vec{n}dS \qquad _v \quad \vec{F}dv$ where $\vec{F} \quad 4xz\hat{i} \quad y^2\hat{j} \quad yz\hat{k}$ and S is the surface of the cube bounded by $x \quad 0$, $x \quad 1$, $y \quad 0$, $y \quad 1$, $z \quad 0$, $z \quad 1$. 6

Unit—III

5. (a) Find the angle through which a set of rectangular axes must be turned without the change of origin so that the expression $7x^2 \quad 4xy \quad 3y^2$ will be transformed into the form $a\,x^2 \quad b\,y^2$. 5

(b) For what value of k will the equation $3x^2 \quad kxy \quad 3y^2 \quad 29x \quad 3y \quad 18 \quad 0$ represent a pair of straight lines? 5

6. (a) Reduce the equation $144x^2 \quad 120xy \quad 25y^2 \quad 243x \quad 448y \quad 113 \quad 0$ to the standard form and hence show that it is the equation of parabola. 6

(b) Find the vertex and length of the latus rectum of the parabola $(3x \quad 4y \quad 17)^2 \quad 35(4x \quad 3y \quad 6)$. 4

Unit—IV

7. *(a)* Find the equation of the plane which passes through the point (2, 3, 1) and is perpendicular to the line joining the points (4, 5, 2) and (2, 1, 6). 4

(b) Find the equation of the plane which passes through the point (2, 1, 4) and is perpendicular to the planes $9x\ \ 7y\ \ 6z\ \ 48\ \ 0$ and $x\ \ y\ \ z\ \ 0$. 4

(c) Find the perpendicular distance of the points (1, 4, 2) and (5, 1, 3) from the plane $2x\ \ 3y\ \ z\ \ 5$. 2

8. *(a)* Prove that the lines

$$\frac{x\ \ 2}{3}\quad\frac{y\ \ 1}{2}\quad\frac{z\ \ 4}{5}$$

and $2x\ \ 3y\ \ z\ \ 0$, $x\ \ y\ \ 2z\ \ 4\ \ 0$ are coplanar. 5

(b) Prove that the shortest distance between the lines

$$\frac{x\ \ 3}{1}\quad\frac{y\ \ 4}{1}\quad\frac{z\ \ 1}{3}\ \text{and}\ \frac{x\ \ 1}{1}\quad\frac{y\ \ 3}{3}\quad\frac{z\ \ 1}{2}\ \text{is}\ \frac{15}{\sqrt{138}}$$

and the equations of the shortest distance are $7x\ \ 37y\ \ 10z\ \ 117\ \ 0$ and $5x\ \ 13y\ \ 17z\ \ 27\ \ 0$. 5

Unit—V

9. *(a)* The plane $\frac{x}{a}\ \ \frac{y}{b}\ \ \frac{z}{c}\ \ 1$ cuts the axes at A, B and C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC. 5

(b) Find the equation of the sphere which passes through the origin and touches the sphere $x^2\ \ y^2\ \ z^2\ \ 56$ at the point (2, 4, 6). 5

10. *(a)* Determine the angle between the lines of intersection of the plane $x \quad 3y \quad z \quad 0$ and the quadric cone $x^2 \quad 5y^2 \quad z^2 \quad 0$. 5

(b) Find the equation of the cylinder generated by the lines parallel to the line

$$\frac{x}{1} \quad \frac{y}{2} \quad \frac{z}{1}$$

and intersecting the guiding curve $z \quad 3$ and $x^2 \quad y^2 \quad 4$. 5

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MATH/I/EC/01 **Student's Copy**

2 0 1 8

(CBCS)

(1st Semester)

MATHEMATICS

FIRST PAPER

(Calculus)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The value of $\lim_{x} \dfrac{\sqrt{x}}{\sqrt{x \quad \sqrt{x \quad \sqrt{x}}}}$ is

(a) 1 ☐

(b) 0 ☐

(c) 2 ☐

(d) 1 ☐

2. The nth derivative of $5x^n$ is

(a) $5n(n \quad 1)^{x^{n \quad 1}}$ ☐

(b) $(5n)!$ ☐

(c) $5n!$ ☐

(d) $5x(n \quad 1)!$ ☐

3. If $f\ (x)\quad 0$ at $x\quad c$, i.e., $f\ (c)\quad 0$, then

(a) $x\quad c$ is the critical point ☐

(b) $f(x)$ is neither increasing nor decreasing at $x\quad c$ ☐

(c) $f(x)$ is decreasing at $x\quad c$ ☐

(d) $f(x)$ is increasing at $x\quad c$ ☐

4. $f(x)\quad e^{x}$ can be expanded in power of $(x\quad 3)$ by using

(a) Maclaurin's theorem ☐

(b) Rolle's theorem ☐

(c) Taylor's theorem ☐

(d) Leibniz's theorem ☐

5. The value of $\displaystyle \ _{0}^{\overline{2}} \sin^4 x \cos^5 x\, dx$ is

(a) $\dfrac{8}{315}$ ☐

(b) $\dfrac{315}{8}$ ☐

(c) $\dfrac{8}{315}$ ☐

(d) $\dfrac{315}{8}$ ☐

6. The value of $\displaystyle \ _{0} \frac{x \sin x}{1\quad \cos^2 x}\, dx$ is

(a) $\dfrac{}{4}$ ☐

(b) $\dfrac{}{2}$ ☐

(c) $\dfrac{2}{2}$ ☐

(d) $\dfrac{2}{4}$ ☐

7. Let $f(x, y) \quad \frac{x \quad y}{x^2 \quad y^2}$, $(x, y) \quad (0, 0)$. Then the lim $f(x, y)$ as $(x, y) \quad (1, 2)$ along the line $y \quad 2$ is

(a) 2 ☐

(b) 1 ☐

(c) Does not exist ☐

(d) None of the above ☐

8. The value of $\quad {}_{0\ 0}^{\overline{2}\ 1} \; y \sin x \, dy \, dx$ is

(a) $\frac{}{2}$ ☐

(b) $\frac{1}{2}$ ☐

(c) 0 ☐

(d) 1 ☐

9. The series $1 \quad x \quad \frac{x^2}{2!} \quad \frac{x^3}{3!} \quad \frac{x^4}{4!} \quad \cdots$

(a) converges only if $x \quad 1$ ☐

(b) converges only if $x \quad 1$ ☐

(c) converges only if $|x| \quad 1$ ☐

(d) converges for all values of x ☐

10. Every Cauchy sequence must be

(a) monotonic ☐

(b) bounded above only ☐

(c) bounded below only ☐

(d) bounded ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. *(a)* A function

$$f(x) \quad x\sin\frac{1}{x} \text{ for } x \quad 0,\ f(0) \quad 0$$

Show that f (0) does not exist.

OR

(b) If the area of a circle increases at a uniform rate, show that the rate of increase of the perimeter varies inversely as the radius.

2. *(a)* Let $f(x) \quad A \quad Bx \quad Cx^2$ in $[a, b]$. Show that the value of c in the mean value theorem f (c) $\frac{f(b) \quad f(a)}{b \quad a}$ is $\frac{a \quad b}{2}$.

OR

(b) Verify Rolle's theorem for $f(x) \quad \frac{1}{x} \quad \frac{1}{1 \quad x}$ in [0, 1].

3. *(a)* Integrate :

$$e^x \frac{1 \quad \sqrt{1 \quad x^2}\sin^{\ 1} x}{\sqrt{1 \quad x^2}} \ dx$$

OR

(b) Evaluate :

$$\int_0^{\overline{4}} \tan^8 x\, dx$$

4. *(a)* If $u \quad \sin^{\ 1} \frac{x^2 \quad y^2}{x \quad y}$, show that $x\frac{u}{x} \quad y\frac{u}{y} \quad \tan u$.

OR

(b) Evaluate the line integral $_C(x^2dx \quad xydy)$ taken along the line segment from (1, 0) to (0, 1).

5. *(a)* Prove that

$$\lim_{m} \frac{1^m \quad 2^m \quad 3^m \quad \cdots \quad n^m}{n^{m \quad 1}} \quad \frac{1}{m \quad 1}; \; m \quad 1$$

OR

(b) Show that a necessary condition for convergence of an infinite series $\quad u_n$ is that $\lim_{n} u_n \quad 0$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Draw the graph of the function defined by

$$f(x) \quad \begin{matrix} x^2 & , \text{ when } x \quad 0 \\ x & , \text{ when } 0 \quad x \quad 1 \\ \frac{1}{x} & , \text{ when } x \quad 1 \end{matrix}$$

Discuss whether $f(x)$ is continuous at $x \quad 1$. 5

(b) Use L' Hospital rule to evaluate

$$\lim_{x \quad 0} \frac{1}{\sin x} \quad \frac{1}{x^3}$$ 5

2. *(a)* Using - , definition of continuity, prove that

$$f(x) \quad \begin{matrix} x^2 \cos(\frac{1}{x}) & , \text{ when } x \quad 0 \\ 0 & , \text{ when } x \quad 0 \end{matrix}$$

is continuous at $x \quad 0$. 4

(b) If $y \quad \frac{\sin^{\quad 1} x}{\sqrt{1 \quad x^2}}$, then prove that

$$(1 \quad x^2) y_{n \quad 2} \quad (2n \quad 3) x y_{n \quad 1} \quad (n \quad 1)^2 y_n \quad 0$$ 6

UNIT—II

3. *(a)* State and prove Rolle's theorem. 5

(b) Expand $\sin x$ in power of $x \quad \frac{}{4}$. 5

4. *(a)* State and prove Taylor's theorem. 6

(b) Let f be defined and continuous on $[a \quad h, a \quad h]$ and derivable on $]a \quad h, a \quad h[$. Prove that there is a real number between 0 and 1 for which $f(a \quad h) \quad f(a \quad h) \quad h\{f \ (a \quad h) \quad f \ (a \quad h)\}$. 4

UNIT—III

5. *(a)* Evaluate : 5

$$\frac{x^2}{(x \quad 1)^2(x \quad 2)}dx$$

(b) Use the definition of the definite integral as a limit of sum to evaluate

$$\frac{1}{x}dx \quad (\text{limits } 1 \text{ to } 3)$$ 5

6. *(a)* If $I_n \quad {}_0^{\overline{2}} x^n \sin x \, dx$, n being a positive integer greater than 1, then obtain the reduction formula for I_n and hence evaluate

$${}_0^{\overline{2}} x^5 \sin x \, dx$$ 6

(b) If $I_n \quad {}_0^{\overline{4}} \tan^n x \, dx$, then prove that

$$n(I_{n \ 1} \quad I_{n \ 1}) \quad 1$$ 4

UNIT—IV

7. *(a)* Let $f : R^2 \quad R$ be a function given by

$$f(x, y) \quad \begin{matrix} \dfrac{x^2 y}{x^4 \quad y^2}, & (x, y) \quad (0, 0) \\ 0 & , (x, y) \quad (0,0) \end{matrix}$$

Check the continuity $f(x, y)$ at $(0, 0)$. 5

(b) If $u \quad \log\sqrt{x^2 \quad y^2 \quad z^2}$, prove that

$$(x^2 \quad y^2 \quad z^2) \quad \frac{{}^2 u}{x^2} \quad \frac{{}^2 u}{y^2} \quad \frac{{}^2 u}{z^2} \quad 1$$ 5

8. *(a)* Evaluate $\ _C xy^4 ds$, where C is the right half of the circle, $x^2 \quad y^2 \quad 16$ rotated in the counterclockwise direction. 5

(b) Change the order of integration

$$\ _0^1 \ _{x^2}^{2 \quad x} xy dx\, dy$$ 5

UNIT—V

9. *(a)* Prove that every monotonic bounded sequence is convergent. 5

(b) Show that the sequence $\{S_n\}$, where

$$S_n \quad \frac{1}{n \quad 1} \quad \frac{1}{n \quad 2} \quad \cdots \quad \frac{1}{n \quad n}, \quad n \quad N$$

is convergent. 5

10. *(a)* Test the convergence of the series

$$\frac{1}{n^{1 \ \frac{1}{n}}}$$ 5

(b) Show that the series

$$\frac{1}{1^p} \quad \frac{1}{2^p} \quad \frac{1}{3^p} \cdots$$

is divergent for $p \quad 1$. 5

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2 0 1 8

(Pre-CBCS)

(1st Semester)

MATHEMATICS

FIRST PAPER

(Calculus)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Tick ☑ the correct answer in the box provided :

1. $\lim\limits_{x \quad 0} \dfrac{a^x \quad 1}{x}$ is equal to

(a) 0 ☐

(b) $\dfrac{1}{x}$ ☐

(c) $\log a$ ☐

(d) Does not exist ☐

2. Differentiation of $\tan^{\ 1} \dfrac{2x}{1 \quad x^2}$ with respect to $\sin^{\ 1} \dfrac{2x}{1 \quad x^2}$ is

(a) 0 ☐

(b) 1 ☐

(c) 2 ☐

(d) None of the above ☐

3. The value of c which is a critical point of mean value theorem for the function $f(x) \quad x(x \quad 1)$ in $[0, 1]$ is

(a) $\frac{1}{2}$ ☐ (b) 1 ☐

(c) 2 ☐ (d) $\frac{1}{4}$ ☐

4. If a curve has a tangent $2y \quad x$ at the origin, then the equation of normal at the origin will be equal to

(a) $2y \quad x \quad 0$ ☐ (b) $y \quad 2x \quad 0$ ☐

(c) $y \quad 2x \quad 0$ ☐ (d) $x \quad y \quad 0$ ☐

5. The definite integral $_0^2 |x \quad 1| dx$ has value

(a) 1 ☐ (b) 2 ☐

(c) 0 ☐ (d) None of the above ☐

6. The value of the integral $_0^{/2} \sin^8 x\, dx$ is equal to

(a) $\frac{7}{256}$ ☐ (b) $\frac{5}{256}$ ☐

(c) $\frac{}{35}$ ☐ (d) $\frac{35}{256}$ ☐

7. If $f(x, y) \quad x^3 y \quad e^x y^2$, then the value of the partial derivative f_{xx} is

(a) $6xy \quad e^x y^2$ ☐ (b) $6y \quad e^x y^2$ ☐

(c) $xy \quad e^x y^2$ ☐ (d) $6x \quad e^x y^2$ ☐

8. If $f(x, y) \quad 2x^2 \quad xy \quad 2y^2$, then the value of $\frac{f}{y}$ at the point $(1, 2)$ is

(a) 5 ☐ (b) 6 ☐

(c) 7 ☐ (d) 8 ☐

9. The geometric series $1 \quad x \quad x^2 \quad x^3 \quad \ldots$ is oscillating infinitely, if

(a) $x \quad 1$ ☐ (b) $|x| \quad 1$ ☐

(c) $x \quad 1$ ☐ (d) None of the above ☐

10. If an infinite series is convergent, then the sequence of its partial sum always

(a) converges to a finite number ☐

(b) converges to 1 ☐

(c) converges to 0 ☐

(d) diverges infinity ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer *all* questions :

1. Evaluate $\lim_{x \to 0} \dfrac{e^x - e^{\sin x}}{x - \sin x}$.

2. Discuss the applicability of Rolle's theorem, when $f(x) = (x-1)(2x-3)$, where $x \in [1, 3]$.

3. Find y_n, when $y = x^2 e^{ax}$.

4. If $U = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$, then prove that $x \dfrac{\partial U}{\partial x} + y \dfrac{\partial U}{\partial y} = \sin 2U$.

5. Prove that every convergent sequence is bounded.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Sketch the graph of the function $y = \dfrac{|x| + x}{2}$ in the domain $[-3, 3]$. Determine the derivability of this function at $x = 0$. 5

(b) Use ϵ-δ definition of continuity to prove that $y = \cos x$ is continuous at every value of x. 5

2. *(a)* Evaluate $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x}$. 5

(b) If $y = \sin^{-1} x$, then show that $(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$. 5

UNIT—II

3. *(a)* State and prove Lagrange's mean value theorem. 5

(b) Find the Maclaurin's series for $\sin x$. 5

4. (a) Verify Rolle's theorem for the function $f(x) \quad x^3 \quad 3x^2 \quad 24x \quad 80$ in $[\quad 4,\ 5]$. 5

(b) State and prove Taylor's theorem. 5

UNIT—III

5. (a) Evaluate $\dfrac{\cot x}{\sin^2 x \quad 3\sin x \quad 1}\,dx$. 5

(b) Evaluate : 5

(i) $\dfrac{dx}{x^2 \quad a^2}$

(ii) $\dfrac{dx}{x(x \quad 1)(x \quad 2)}$

6. (a) Evaluate $\,{}_0^2\, x^3\,dx$. 4

(b) If n be a positive integer and let

$$I_n \quad {}_0^{/2} \sin^n x\,dx$$

then show that

$$I_n \quad \frac{(n \quad 1)(n \quad 3)\ldots 3 \quad 1}{n(n \quad 2)\ldots 4 \quad 2} \quad \overline{2}$$

if n is even. 6

UNIT—IV

7. (a) If $V \quad \log \dfrac{x^3 \quad y^3}{x^2 \quad y^2}$, then prove that

$$x\frac{V}{x} \quad y\frac{V}{y} \quad 1$$

5

(b) Discuss the continuity of $f(x,\ y)$ at the origin, where

$$f(x,\ y) \quad \begin{matrix} \dfrac{x^3 \quad y^3}{x^2 \quad y^2} & ,\ (x,\ y) \quad (0,\ 0) \\ 0 & ,\ (x,\ y) \quad (0,\ 0) \end{matrix}$$

5

8. *(a)* Find the area bounded by the curves $x^2 \quad y^2 \quad 9$ and $y \quad 2\sqrt{2}x$ in the first quadrant only. 5

(b) Evaluate $\quad xy\,dxdy$ over the region in the positive quadrant in which $x \quad y \quad 1$. 5

UNIT—V

9. *(a)* Define Cauchy sequence and prove that a sequence $\{S_n\}$ converges iff it is Cauchy sequence. 5

(b) Show the sequence $\{a_n\}$, where $a_n \quad \dfrac{3n \quad 1}{n \quad 2}$ is convergent. 5

10. *(a)* Prove that the positive term series $\quad \dfrac{1}{n^p}$ is convergent, if and only if $p \quad 1$. 5

(b) Examine the convergence of $\displaystyle \ _{n \quad 1}\frac{1}{n!}$. 5

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